



Class 11 Maths || Chapter 3 : Trigonometric Functions || Class Notes

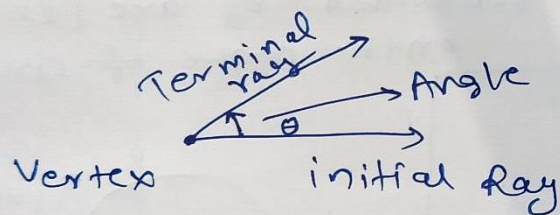
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Class - 11 Maths

Chapter - 3 Trigonometric Functions

Exercise 3.1 * Basics of Angles

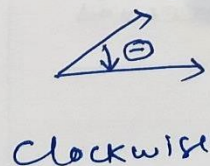
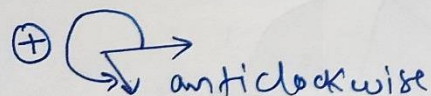
Angles: measurement of rotation between two arms (rays)



Angle

Positive angle

Negative Angle

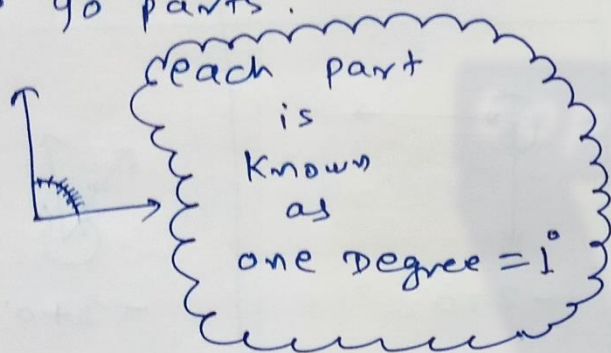


 $+ 30^\circ$	 $- 120^\circ$
 $- 270^\circ$	 $- 340^\circ$
 $+ 360^\circ$	 $- 360^\circ$

One Complete revolution

Degree measurement:

A right angle is divided into 90 parts.



$$1^\circ = 60 \text{ parts} = 60 \text{ minutes} = 60'$$

$$1 \text{ minute} = 60 \text{ parts} = 60 \text{ seconds} = 60''$$

$$1 \text{ right angle} = 90^\circ = (90 \times 60)' = (90 \times 60 \times 60)''$$

↑
General
Language

↑
Degree

↑
minute

↑
seconds

Radian Measurement:

[Circular System]

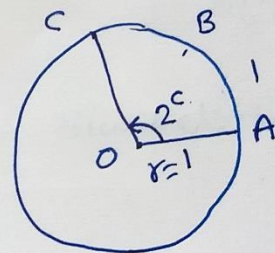
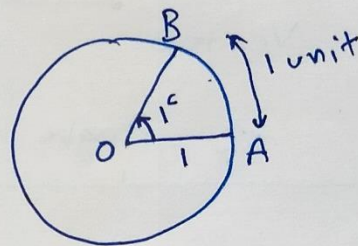
$$1 \text{ radian} = 1^c = 1$$

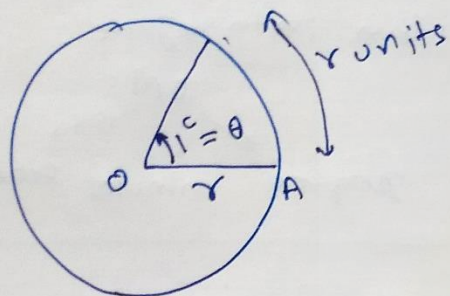
Definition of unit circle

$$r = 1 \text{ unit}$$

Definition of one radian

In a circle ($r=1 \text{ unit}$), angle subtended by arc ($l=1 \text{ unit}$) on the centre of circle =





$$\theta = \frac{l}{r}$$

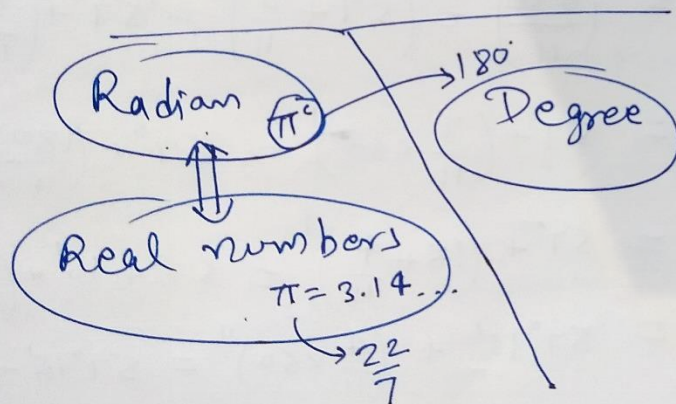
length of arc

radius of circle

Angle subtended by arc on the centre

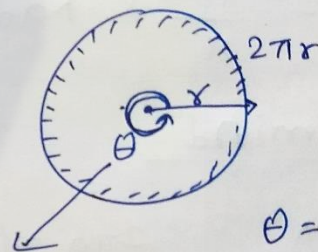
(only in radian)

Note :



Relation b/w Degree & Radian

one complete revolution = $360^\circ = 2\pi^c$



$$\theta = \frac{l}{r}$$

$$\theta = \frac{2\pi r}{r} = 2\pi$$

$$\theta = 2\pi = 2\pi \text{ radian} = 2\pi^c$$

So Relation is

$$360^\circ = 2\pi^c$$

$$270^\circ = \left(\frac{3\pi}{2}\right)^c$$

$$180^\circ = \pi^c$$

$$90^\circ = \left(\frac{\pi}{2}\right)^c$$

$$45^\circ = \left(\frac{\pi}{4}\right)^c$$

$$1^\circ = \left(\frac{2\pi}{360}\right)^c$$

$$270^\circ = \left(\frac{\pi}{180}\right) \cdot 270$$

↑ Degree ↑ Rad.

Formula For Conversion

$$\text{Radian} = \frac{\pi}{180} \times \text{Degree}$$

$$\text{Degree} = \frac{180}{\pi} \times \text{Radian}$$

Keep in mind

$$360^\circ = 2\pi^c$$

$$180^\circ = \pi^c$$

$$150^\circ = \left(\frac{5\pi}{6}\right)^c$$

$$90^\circ = \left(\frac{\pi}{2}\right)^c$$

$$45^\circ = \left(\frac{\pi}{4}\right)^c$$

$$270^\circ = \left(\frac{3\pi}{2}\right)^c$$

$$120^\circ = \left(\frac{2\pi}{3}\right)^c$$

$$135^\circ = \left(\frac{3\pi}{4}\right)^c$$

$$60^\circ = \left(\frac{\pi}{3}\right)^c$$

$$30^\circ = \left(\frac{\pi}{6}\right)^c$$

$$\begin{array}{r} 11 \overline{) 630} \quad (57 \\ \underline{ 57} \\ 60 \\ \underline{ 57} \\ 30 \\ \underline{ 30} \\ 0 \end{array}$$

e.g. Convert 1 radian into degree.

DMS
↙ ↓ ↘
Degree minutes seconds

Ans. 1 radian = ?? Degree

$$\text{Degree} = \frac{180}{\pi} \times \text{radian}$$

$$D = \left(\frac{180}{\left(\frac{22}{7}\right)} \times 1 \right)^\circ = \left(\frac{180 \times 7}{22} \right)^\circ = \left(\frac{90 \times 7}{11} \right)^\circ$$

$$D = \left(\frac{630}{11} \right)^\circ = \left(57 + \frac{3}{11} \right)^\circ = 57^\circ + \left(\frac{3}{11} \right)^\circ$$

$$= 57^\circ + \left(\frac{3}{11} \times 60 \right)' = 57^\circ + \left(\frac{180}{11} \right)'$$

$$= 57^\circ + \left(16 + \frac{4}{11} \right)' = 57^\circ + 16' + \left(\frac{4}{11} \right)'$$

$$= 57^\circ 16' + \left(\frac{4}{11} \times 60 \right)'' = 57^\circ 16' + \frac{24}{11}''$$

$$= 57^\circ 16' 21'' \checkmark$$

approx.

e.g.

Convert $40^{\circ}30'$ into radian.

$$\text{radian} = \frac{\pi}{180} \times \underline{\text{Degree}}$$

$$= \frac{\pi}{180} \times (40^{\circ}30')$$

$$= \frac{\pi}{180} \times \left(40^{\circ} + \frac{30^{\circ}}{60}\right)$$

$$= \frac{\pi}{180} \times \left(40 + \frac{1}{2}\right)^{\circ}$$

$$= \frac{\pi}{180} \times \left(\frac{81}{2}\right)^{\circ}$$

$$= \frac{\pi \times 9}{20 \times 2} = \frac{9\pi}{40} \text{ radian}$$

$$\underline{40^{\circ}30' = \frac{9\pi}{40} \text{ radian}}$$

radian π
 $\downarrow$
Degree 180°

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Exercise 3.1

①

(i) $25^\circ = \frac{5\pi}{36}$

Radian = $\frac{\pi}{180} \times \text{Degree}$

$= \frac{\pi}{180} \times 25^\circ$

$= \frac{5\pi}{36}$

$\frac{5\pi}{36}$

(ii) $-(47^\circ 30')$

~~⊗~~ $= -(47^\circ + (\frac{30'}{60}))$

$= -(47 + \frac{1}{2})^\circ$

$= -(\frac{95}{2})^\circ$

Radian = $\frac{\pi}{180} \times (\frac{-95}{2})^\circ$
 $= \frac{-19\pi}{72}$

(iii) 240°

Radian = $\frac{\pi}{180} \times 240^\circ = \frac{4\pi}{3}$

(iv) 520°

Radian = $\frac{\pi}{180} \times 520^\circ = \frac{26\pi}{9}$

$$(2) (i) \left(\frac{11}{16}\right)^c$$

$$\text{Degree} = \left(\frac{180}{\pi} \times \text{Radian}\right)$$

$$\left(\pi = \frac{22}{7}\right)$$

$$= \frac{180}{\left(\frac{22}{7}\right)} \times \frac{11}{16}$$

$$= \frac{180}{22} \times 7 \times \frac{11}{16}$$

$$= \left(\frac{315}{8}\right)^0$$

$$= \left(39 \frac{3}{8}\right)^0$$

$$= 39^\circ \left(\frac{3}{8}\right)^0 = 39^\circ + \left(\frac{3}{8} \times \frac{15}{2}\right)'$$

$$\begin{array}{r} 8 \overline{) 315} \quad (39) \\ \underline{24} \\ 75 \\ \underline{72} \\ 3 \end{array}$$

$$\Rightarrow 39^\circ + \left(\frac{45}{2}\right)'$$

$$= 39^\circ + \left(22 \frac{1}{2}\right)'$$

$$= 39^\circ + 22' + \left(\frac{1}{2}\right)''$$

$$= 39^\circ + 22' + \left(\frac{1}{2} \times 60\right)''$$

$$= 39^\circ + 22' + 30''$$

$$= 39^\circ 22' 30''$$

$$(ii) (-4)^c$$

$$\pi = \frac{22}{7}$$

$$\text{Degree} = \frac{180}{\pi} \times (-4)$$

$$= \frac{180}{22} \times 7 \times (-4)$$

$$= - \frac{1260}{11} \times (2)$$

$$= - \left(\frac{2520}{11}\right)^0$$

$$\begin{aligned}
 &= -\left(\frac{2520}{11}\right)^\circ \\
 &= -\left(229 \frac{1}{11}\right)^\circ \\
 &= -\left(229^\circ + \left(\frac{1}{11}\right)^\circ\right) \\
 &= -\left(229^\circ + \left(\frac{60}{11}\right)'\right) \\
 &= -\left(229^\circ + \left(5 + \frac{5}{11}\right)'\right) \\
 &= -\left(229^\circ + 5' + \left(\frac{5}{11}\right)'\right) \\
 &= -\left(229^\circ + 5' + \left(\frac{5}{11} \times 60\right)''\right) \\
 &= -\left(229^\circ + 5' + \left(\frac{300}{11}\right)''\right) \\
 &= -\left(229^\circ + 5' + 27''\right) \\
 &= -\underline{229^\circ 5' 27''}
 \end{aligned}$$

③

In one minute $\Rightarrow$ 360 revⁿ.

$\Rightarrow$ 60 sec. $\longrightarrow$ 360 revⁿ

1 sec. $\longrightarrow$ $\left(\frac{360}{60}\right)$ revⁿ

1 sec. $\longrightarrow$ 6 revⁿ

$= (6 \times 2\pi)$ radian
 $\rightarrow = \underline{(12\pi)^c}$

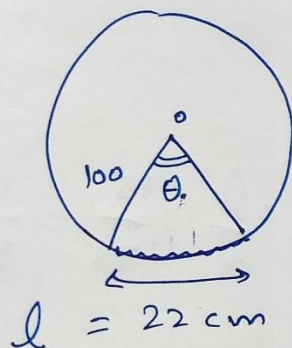


1 revⁿ
 $= (2\pi)^c$

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Exercise 3.1 (class 11 Maths)

(4)



$$r = 100 \text{ cm}$$

radian

$$\theta = \frac{l}{r}$$

$$\theta = \left(\frac{22}{100}\right)^c$$

$$\text{Degree} = \frac{180}{\pi} \times (\text{Radian})$$

$$= \frac{180}{\left(\frac{22}{7}\right)} \times \frac{22}{100}$$

$$= \left(\frac{126}{10}\right)^{\circ}$$

$$= \left(\frac{126}{10}\right)^{\circ}$$

$$= (12.6)^{\circ}$$

$$= 12^{\circ} + \left(\frac{6}{10}\right)^{\circ}$$

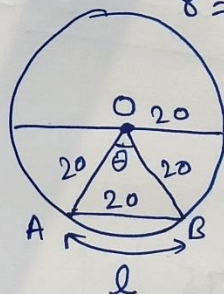
$$= 12^{\circ} + \left(\frac{6}{10} \times 60\right)' = 12^{\circ} + (36)'$$

$$= 12^{\circ} 36'$$

(5)

$$d = 40 \text{ cm}$$

$$r = 20 \text{ cm}$$



$$\widehat{AB} = ?$$

$$l = r\theta$$

radian

$$20$$

$$\theta = 60^{\circ}$$

$\triangle OAB \rightarrow$ Equilateral $\triangle$

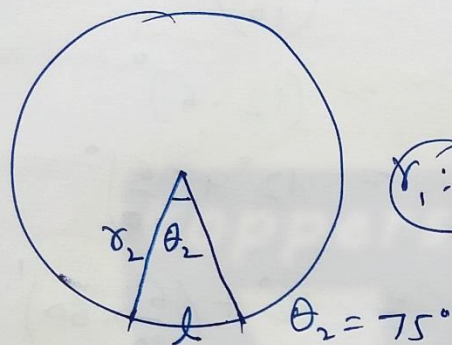
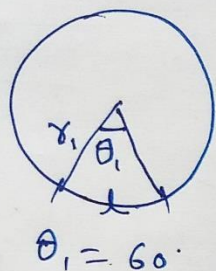
$$\text{minor arc} = l = r\theta$$

$$\Rightarrow l = 20 \times \frac{\pi}{3} = \frac{20\pi}{3}$$

$$\theta = \frac{\pi}{180} \times 60$$

$$= \frac{\pi}{3}$$

⑥



$r_1 : r_2$

length of arc = same = l

$l = r_1 \theta_1$

$l = r_1 \theta_1$

$l = r_2 \theta_2$

$r_1 \theta_1 = r_2 \theta_2$

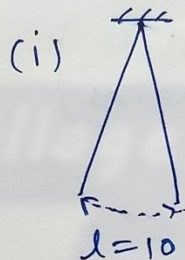
$\Rightarrow r_1 (60) = r_2 (75)$

$\Rightarrow \frac{r_1}{r_2} = \frac{75}{60} = \frac{5}{4}$

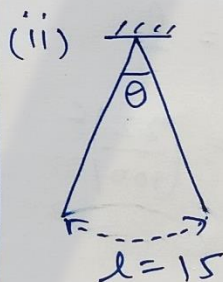
$r_1 : r_2 = 5 : 4$

⑦

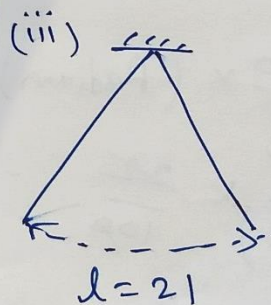
$r = 75 \text{ cm}$ radian
 $\theta = ?$



radian
 $\theta = \frac{l}{r} = \frac{10}{75} = \frac{2}{15}$



$\theta = \frac{l}{r} = \frac{15}{75} = \frac{1}{5}$



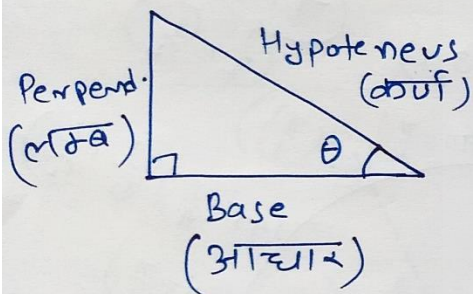
$\theta = \frac{l}{r} = \frac{21}{75} = \frac{7}{25}$

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Theory Before Exercise 3.2

Trigonometric Functions

[Circular Functions]



For English medium →

$\begin{matrix} \sin & \cos & \tan \\ \downarrow & \downarrow & \downarrow \\ P & B & P \\ \hline H & H & B \\ \uparrow & \uparrow & \uparrow \\ \text{Cosec} & \sec & \cot \end{matrix}$

For Hindi medium →

$\begin{matrix} \sin & \cos & \tan \\ \downarrow & \downarrow & \downarrow \\ L & A & L \\ \hline K & K & A \\ \uparrow & \uparrow & \uparrow \\ \text{Cosec} & \sec & \tan \end{matrix}$

Identities

$$P^2 + B^2 = H^2$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sec^2 x = 1 + \tan^2 x \rightarrow \sec x + \tan x = \frac{1}{\sec x - \tan x}$$

$$\text{cosec}^2 x = 1 + \cot^2 x \rightarrow$$

Trigonometric Table

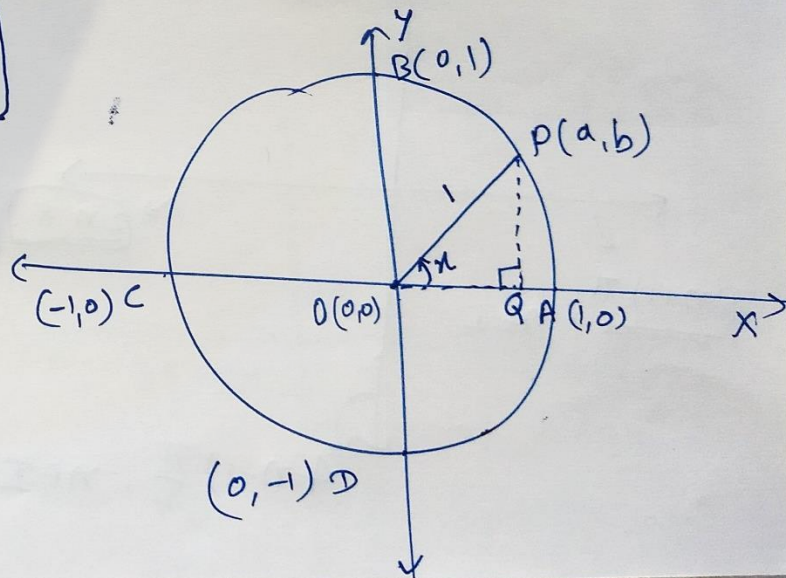
0°, 30°, 45°, 60°, 90°

↑ ↑ till 10th class ↑ ↑

Now class -11, Trigonometric Fnⁿ

Unit Circle
r=1
centre (0,0)

$OP = 1$
 $PQ = b$
 $OQ = a$



In ΔOPQ

$$\sin x = \frac{PQ}{OP} = \frac{b}{1} = b$$

$$\cos x = \frac{OQ}{OP} = \frac{a}{1} = a$$

$$P(a, b) \equiv P(\cos x, \sin x) \quad \star$$

Note
① $OP^2 = OQ^2 + PQ^2$

$$\Rightarrow 1 = a^2 + b^2$$

$$\Rightarrow \boxed{1 = \cos^2 x + \sin^2 x}$$

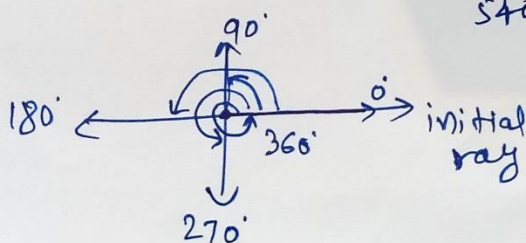
Note

②

Quadrantal Angles

90° multiple = $0, 90, \dots, 360, 450, 540, \dots$

$\frac{\pi}{2}$



$\longleftrightarrow$

π

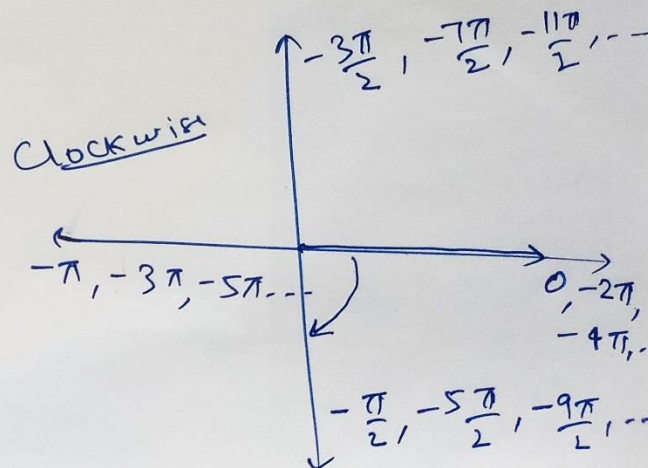
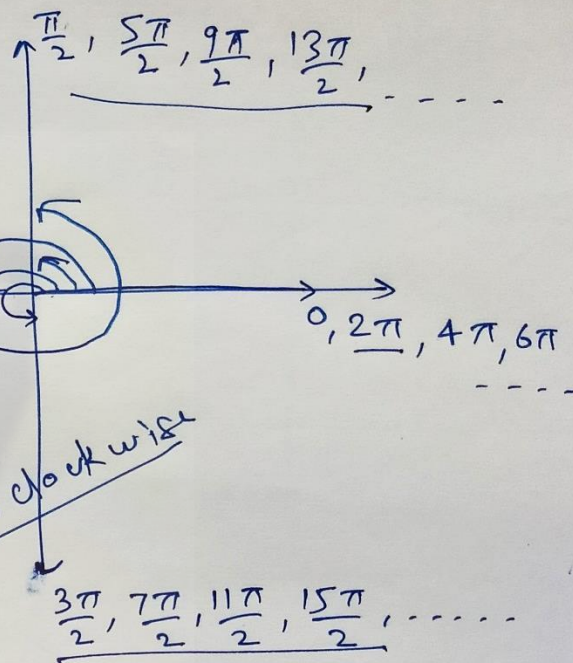
3π

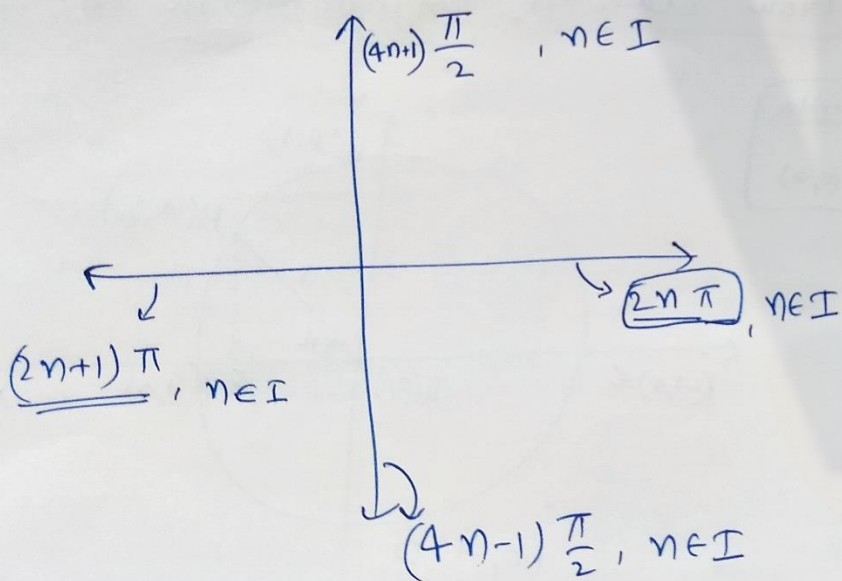
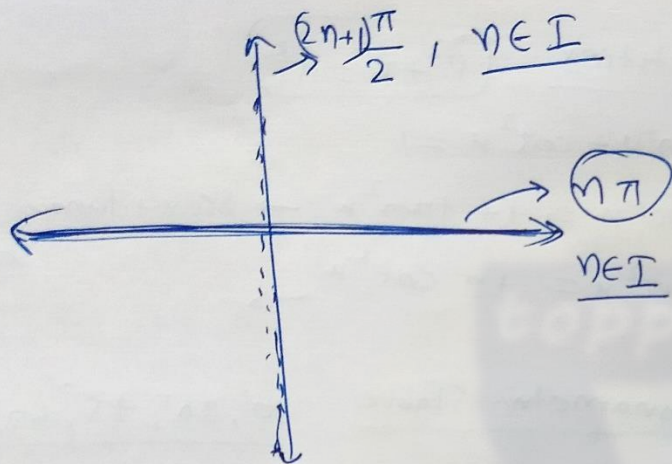
5π

7π

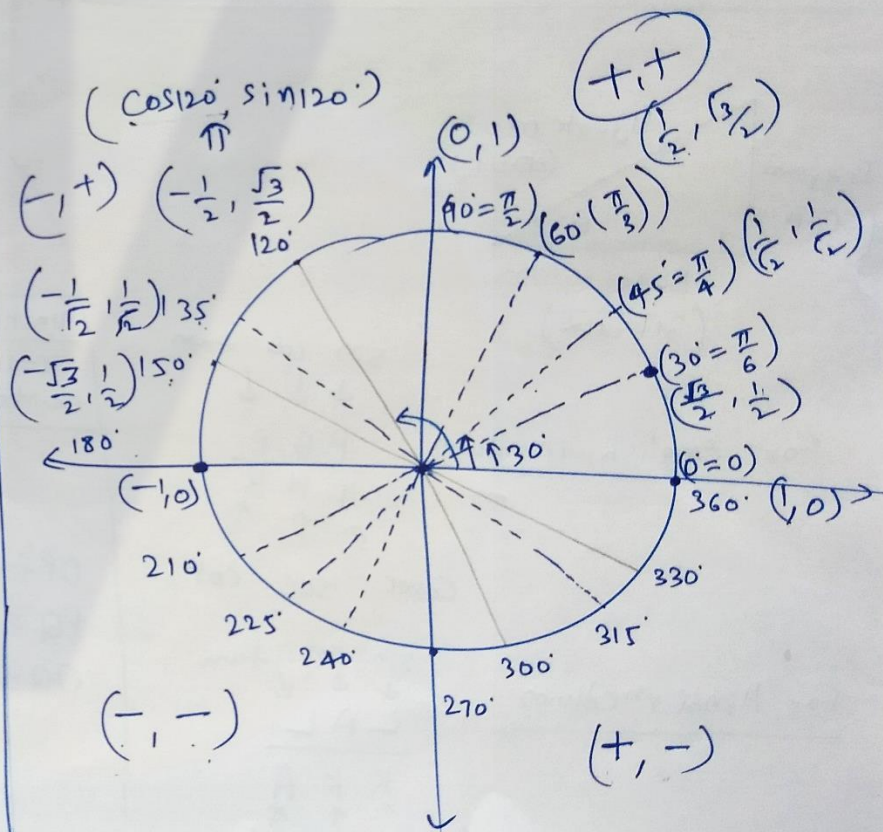
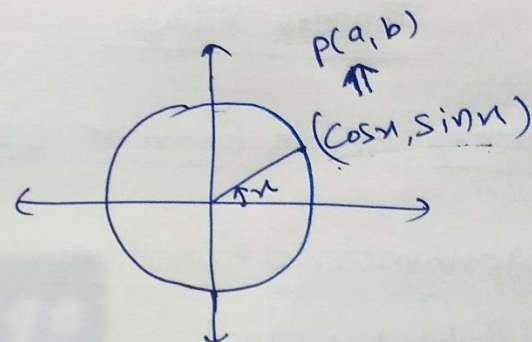
$\vdots$

Anti-clockwise

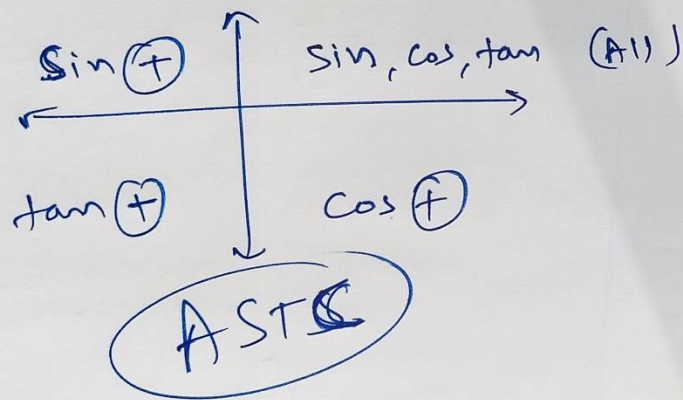
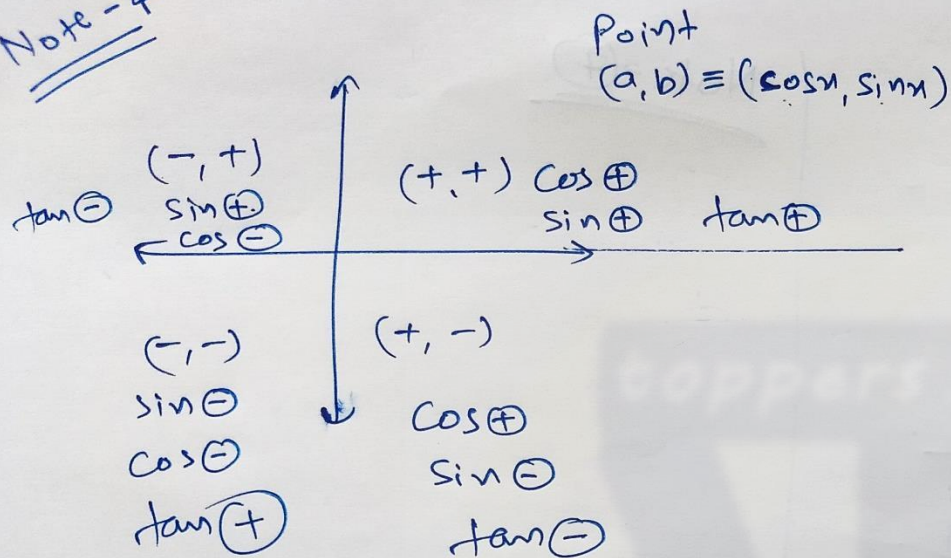




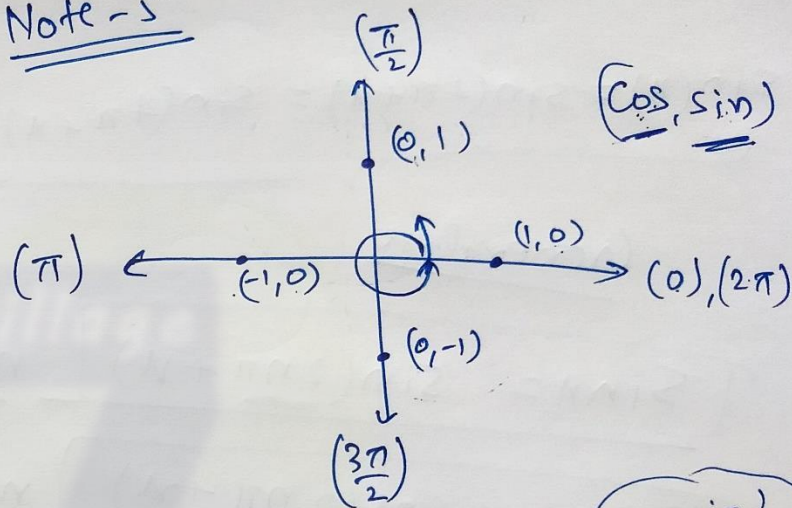
Note
③



Note - 4



Note - 5



$\cos 0 = 1 \leftarrow \sin 0 = 0 \checkmark$
 $\cos \frac{\pi}{2} = 0 \rightarrow \sin \frac{\pi}{2} = 1$
 $\cos \pi = -1 \leftarrow \sin \pi = 0 \checkmark$
 $\cos \frac{3\pi}{2} = 0 \rightarrow \sin \frac{3\pi}{2} = -1$
 $\cos 2\pi = 1 \leftarrow \sin 2\pi = 0 \checkmark$

$\sin 30^\circ = \frac{1}{2}$
 $\sin 390^\circ = \frac{1}{2}$

$\sin(x) = \sin\left(\frac{2\pi}{\uparrow} + x\right)$

One complete revolution

$$\sin(x) = \sin(2\pi + x) = \sin(4\pi + x) \dots$$

Generalize

$$\boxed{\sin x = \sin(2n\pi + x)} \quad n \in \mathbb{I}$$

$$\boxed{\cos x = \cos(2n\pi + x)} \quad n \in \mathbb{I}$$

Note: 6

$$\frac{\sin x = 0}{\boxed{\sin n\pi = 0}}, \quad \boxed{x = n\pi}, \quad n \in \mathbb{I}$$

$$\cos x = 0 \longrightarrow x = \underline{(n+1)\frac{\pi}{2}}, \quad n \in \mathbb{I}$$

$$\longleftarrow \updownarrow \longrightarrow \quad \underline{\cos(2n+1)\frac{\pi}{2} = 0}$$

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Class - 11 - Maths

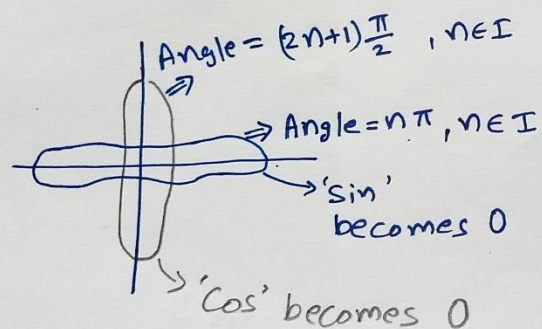
Trigonometric Functions { Before Exercise 3.2 }

From Last Lecture

$\sin \oplus$	All $\oplus$
$\tan \oplus$	$\cos \oplus, \sec \oplus$

Complementary

$\sin \rightarrow \cos$
 $\cos \rightarrow \sin$
 $\tan \rightarrow \cot$
 $\cot \rightarrow \tan$
 $\sec \rightarrow \csc$
 $\csc \rightarrow \sec$



How to find values of trigonometric ratios values for Big Angles??

0, 30, 45, 60, 90

(i) Quadrant $\rightarrow$ Final Answer

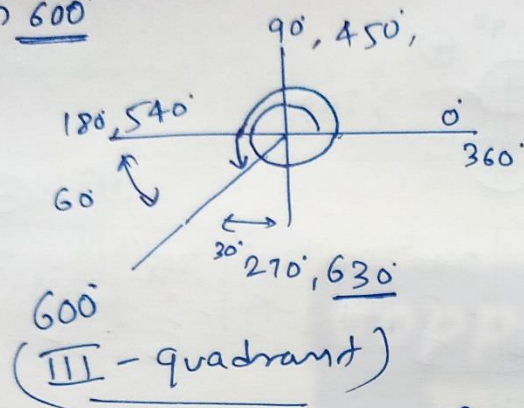
Sign $\rightarrow \oplus$
 $\rightarrow \ominus$

(ii) Angle = $90 \times n \pm \theta \approx \theta$

Division Algorithm

n $\rightarrow$ Even $\rightarrow$ No complementary
 $\rightarrow$ odd $\rightarrow$ Complementary.

e.g. (i) $\sin 600^\circ$



$$90 \overline{) 600} \begin{array}{r} 6 \\ - 540 \\ \hline 60 \end{array}$$

$$600^\circ = 90^\circ \times 6 + 60^\circ$$

$$\sin(\underbrace{600^\circ}_{\text{III}}) = \sin(\underbrace{90^\circ \times 6 + 60^\circ}_{\text{III}})$$

↑
Even

$$= -\sin 60^\circ$$

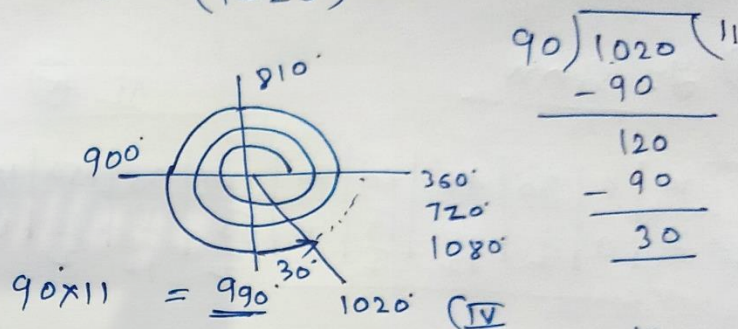
$$= -\frac{\sqrt{3}}{2}$$

$$\sin(600^\circ) = \sin(\underbrace{90^\circ \times 7 - 30^\circ}_{\text{III}})$$

↑
Odd

$$= -\cos(30^\circ) = -\frac{\sqrt{3}}{2}$$

(ii) $\sec(1020^\circ)$



$$90 \overline{) 1020} \begin{array}{r} 11 \\ - 90 \\ \hline 120 \\ - 90 \\ \hline 30 \end{array}$$

$$\sec(1020^\circ) = \sec(90^\circ \times 11 + 30^\circ)$$

↑
Odd

IV

$$= + \operatorname{cosec}(30^\circ)$$

$$\sec(1020^\circ) = +2$$

(iii) $\tan(495^\circ)$

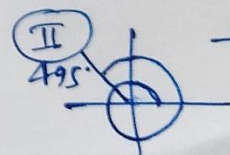
$$= \tan(90^\circ \times 5 + 45^\circ)$$

↑
Odd

$$90 \overline{) 495} \begin{array}{r} 5 \\ - 450 \\ \hline 45 \end{array}$$

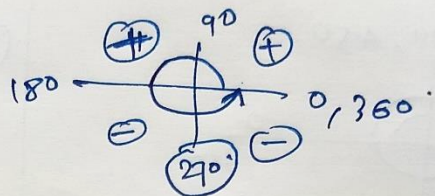
$$= -\cot(45^\circ)$$

$$= -(1)$$

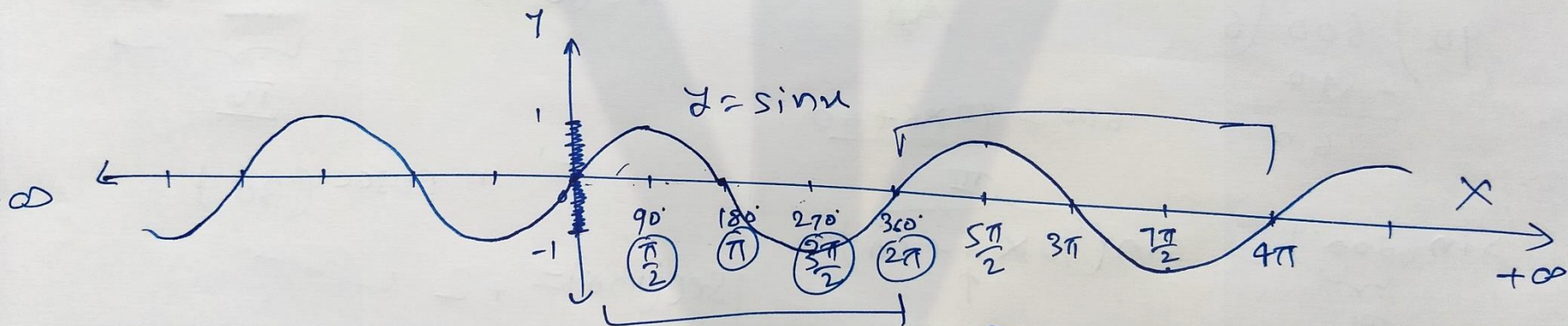


Graph + Domain + Range

① $y = \sin x$



						II (+)										III = (-)			
x	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360		
y = sin x	0	1/2	√2/2	√3/2	1	√3/2	1/2	1/2	0	-1/2	-√2/2	-√3/2	-1	-√3/2	-1/2	-1/2	0		
	I (+)									IV (-)									

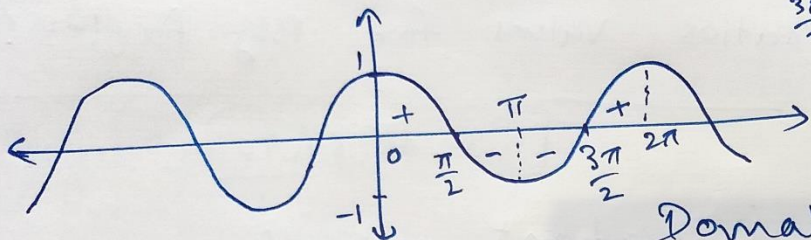
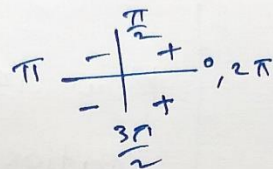


Repeat
 $\leftarrow (2\pi) \rightarrow$

Domain = $(-\infty, \infty) = \mathbb{R}$
 Range = $[-1, 1]$ ✓

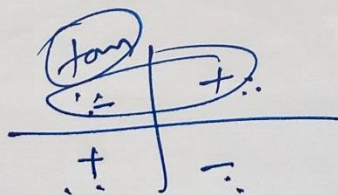
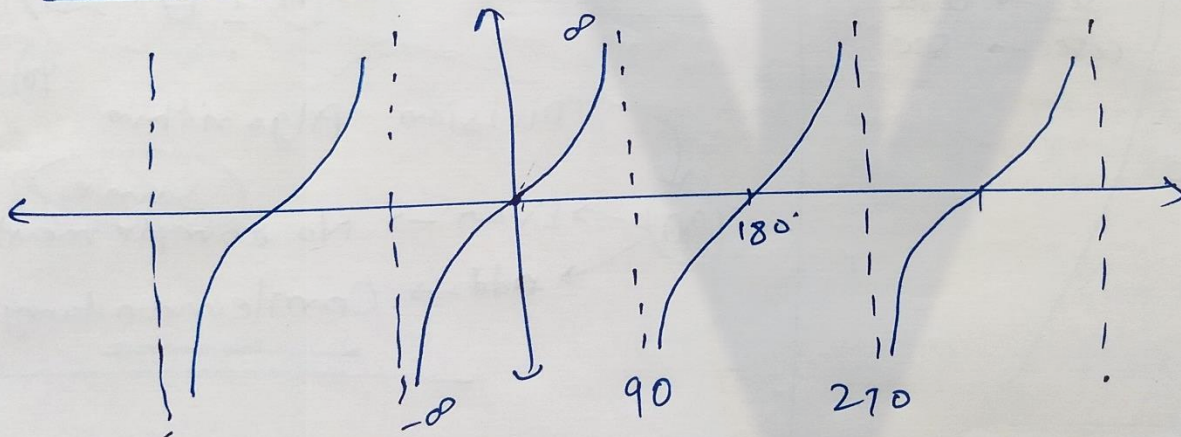
$-1 \leq \sin x \leq 1$ ★

② $y = \cos x$



Domain = $(-\infty, \infty) = \mathbb{R}$
Range = $[-1, 1]$

③ $y = \tan x = \frac{\sin x}{\cos x} \neq 0$ $x \in \mathbb{R} - \left\{ a : a = (n+1)\frac{\pi}{2} \right\}$

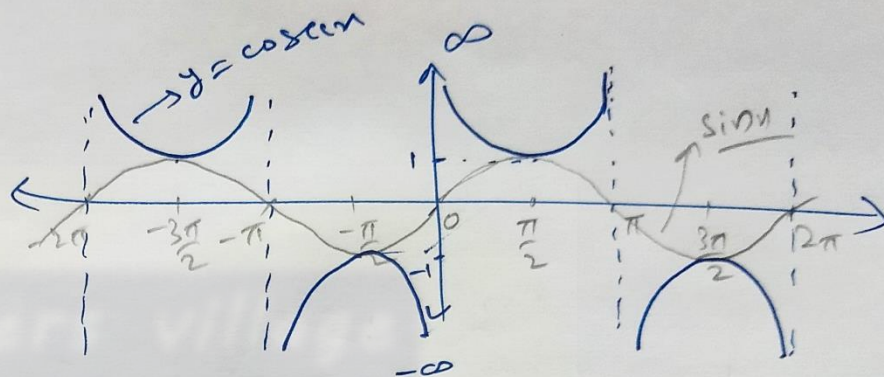


Domain = $\mathbb{R} - \left\{ x : x = (2n+1)\frac{\pi}{2} \right\}$ $n \in \mathbb{I}$
Range = $(-\infty, \infty) = \mathbb{R}$

④ $y = \operatorname{cosec} x = \frac{1}{\sin x} \neq 0$

Domain: $\mathbb{R} - \{x: x = n\pi, n \in \mathbb{I}\}$

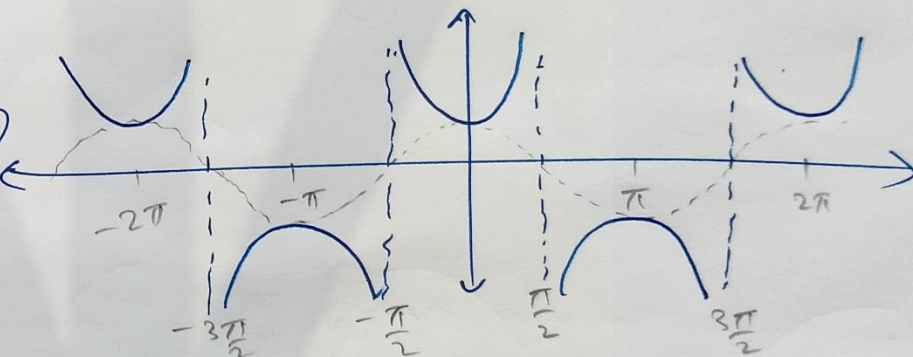
Range: $(-\infty, -1] \cup [1, \infty)$



⑤ $y = \sec x = \frac{1}{\cos x} \neq 0$

Domain: $\mathbb{R} - \left\{x: x = (2n+1)\frac{\pi}{2}\right\}$

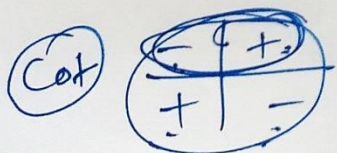
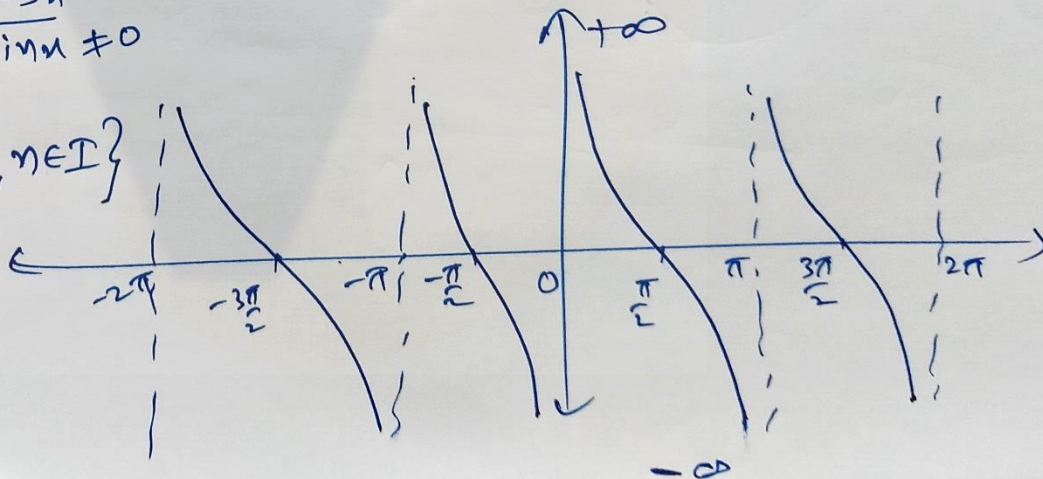
Range: $(-\infty, -1] \cup [1, \infty)$



⑥ $y = \tan x = \frac{\sin x}{\cos x} \neq 0$

Domain: $\mathbb{R} - \{x: x = n\pi, n \in \mathbb{I}\}$

Range: $(-\infty, \infty) = \mathbb{R}$



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Class - 11 - Maths

Exercise 3.2

① $\cos x = -\frac{1}{2}$, $x \rightarrow$ II quadrant

$\sin x, \tan x, \cot x, \sec x, \operatorname{cosec} x$
↓ ⊕ ⊕ ↓ ↓
⊖ ⊖ ⊖

$$\underline{\sin^2 x} + \underline{\cos^2 x} = 1 \quad \checkmark$$

$$\Rightarrow \sin^2 x + \left(-\frac{1}{2}\right)^2 = 1 \quad \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\Rightarrow \sin^2 x = 1 - \frac{1}{4}$$

$$\Rightarrow \sin^2 x = \frac{4-1}{4} = \frac{3}{4}$$

$$\boxed{\sin x = \pm \frac{\sqrt{3}}{2}}$$

+	π	+	I
-	π	-	IV

π (circled) with an arrow pointing to the third quadrant.

$$\sin x = -\frac{\sqrt{3}}{2}, \cos x = -\frac{1}{2}$$

Diagram showing a horizontal line with arrows at both ends. A point is marked on the line, and a vertical line segment of length $\frac{\sqrt{3}}{2}$ is drawn downwards from it. Another vertical line segment of length $\frac{1}{2}$ is drawn downwards from the same point, to the right of the first one.

$$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

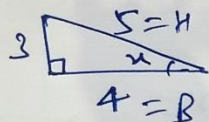
$$\textcircled{2} \sin x = \frac{3}{5}, x \rightarrow \text{II}$$

$\sin$	+
$\tan$	+
$\cos$	-

$$\cos x, \tan x, \cot x, \operatorname{cosec} x, \sec x$$

- - - $\oplus$ -

$$\sin x = \frac{3}{5} = \frac{P}{H}$$



$$\cos x = \frac{B}{H} = -\frac{4}{5}$$

$$\tan x = \frac{P}{B} = -\frac{3}{4}$$

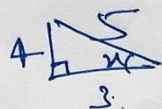
$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{5}{3}$$

$$\sec x = \frac{1}{\cos x} = -\frac{5}{4}$$

$$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$$

$$\textcircled{3} \cot x = \frac{3}{4}, x \rightarrow \text{II}$$

$$\cot x = \frac{3}{4} = \frac{B}{P}$$



$$\sin x = \frac{P}{H} = -\frac{4}{5}$$

$$\cos x = \frac{B}{H} = -\frac{3}{5}$$

$$\tan x = \frac{4}{3}$$

$$\sec x = -\frac{5}{3}$$

$$\operatorname{cosec} x = -\frac{5}{4}$$

$$\textcircled{4} \sec x = \frac{13}{5} = \frac{H}{B}, x \rightarrow \text{IV}$$

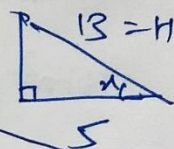
$$\cos x = \frac{5}{13}$$

$$\sin x = \frac{P}{H} = -\frac{12}{13} = -\frac{12}{13}, P = 12$$

$$\tan x = -\frac{12}{5}$$

$$\cot x = -\frac{5}{12}$$

$$\operatorname{cosec} x = -\frac{13}{12}$$



⑤ $\tan u = -\frac{5}{12} = \frac{P}{B}$ $u \rightarrow \text{II}$

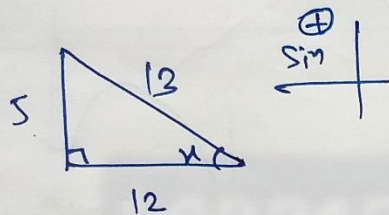
$\cot u = -\frac{12}{5}$

$\sin u = \frac{5}{13}$

$\cos u = -\frac{12}{13}$

$\sec u = -\frac{13}{12}$

$\operatorname{cosec} u = \frac{13}{5}$



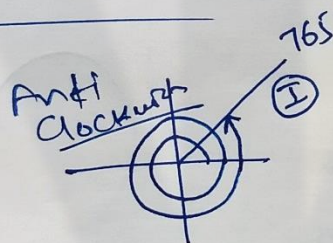
⑥ $\sin(765^\circ)$

$765 = 90 \times \text{II} + \theta$ quadrant

$$\begin{array}{r} 90 \overline{) 765} \quad (8 \\ \underline{-720} \\ 45 \end{array}$$

$765 = 90 \times 8 + 45$

$$\begin{aligned} \sin(765^\circ) &= \sin(90 \times 8 + 45^\circ) \\ &= +\sin(45^\circ) \\ &= \frac{1}{\sqrt{2}} \end{aligned}$$



Even

⑦ $\operatorname{cosec}(-1410^\circ)$

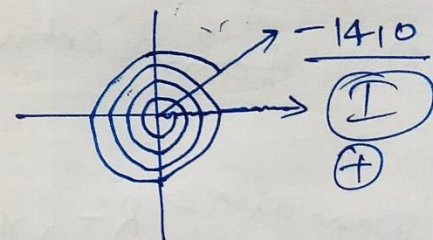
$90 \times \text{II} + \theta$ quadrant

$$\begin{array}{r} 90 \overline{) 1410} \quad (15 \\ \underline{-90} \\ 510 \\ \underline{-450} \\ 60 \end{array}$$

$1410 = 90 \times 15 + 60$

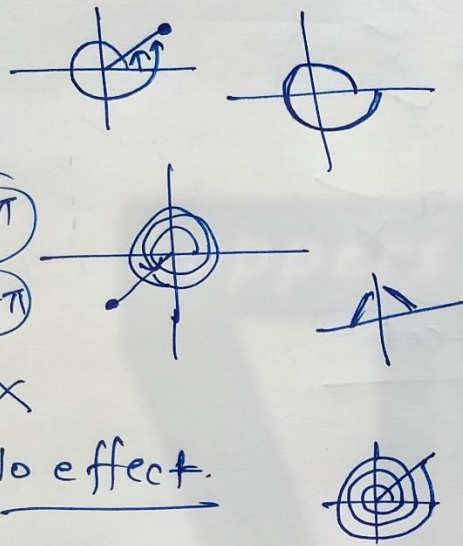
Angle = $-1410 = -90 \times 15 - 60$

Clock



$$\begin{aligned} \operatorname{cosec}(-1410^\circ) &= \operatorname{cosec}(-90 \times 15 - 60^\circ) \\ &= +\sec(60^\circ) \\ &= 2 \end{aligned}$$

⑧ $\tan\left(\frac{19\pi}{3}\right)$



$\begin{aligned} + 360^\circ &\rightarrow 2\pi \\ + 720^\circ &\rightarrow 4\pi \\ + 1080^\circ &\rightarrow 6\pi \end{aligned}$

No effect.

$$\begin{aligned} \tan\left(\frac{19\pi}{3}\right) &= \tan\left(\underline{6\pi} + \frac{\pi}{3}\right) \\ &= \tan\left(\frac{\pi}{3}\right) \\ &= \sqrt{3} \quad \checkmark \end{aligned}$$

⑨ $\sin\left(-\frac{11\pi}{3}\right)$

$\xrightarrow{\text{Degree}} \frac{\pi \rightarrow 180^\circ}{\text{Radian}}$

$= \sin\left(-\frac{11 \times 180^\circ}{3}\right)$

$= \sin(-660^\circ)$

$= \sin(+\underline{720^\circ} - 660^\circ)$

$720^\circ = \underline{2 \times 360^\circ}$

$$\begin{aligned} &= \sin(60^\circ) \\ &= \frac{\sqrt{3}}{2} \quad \checkmark \end{aligned}$$

$$(10) \cot\left(-\frac{15\pi}{4}\right)$$

$$= \cot\left(\frac{-16\pi + \pi}{4}\right)$$

$$= \cot\left(-\frac{16\pi}{4} + \frac{\pi}{4}\right)$$

$$= \cot\left(\cancel{-4\pi} + \frac{\pi}{4}\right)$$

$$= \cot\left(\frac{\pi}{4}\right)$$

$$= \cot(45^\circ)$$

$$= 1$$

$$\begin{array}{l} \pm 2\pi \quad \cancel{x} \\ \pm 4\pi \quad \cancel{x} \\ \pm 6\pi \quad \cancel{x} \\ \pm (\text{even})\pi \quad \cancel{x} \end{array}$$

no effect-

Remove,

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chapter: 3 Trigonometric Functions (Class II)

All formulae (Before Exercise 3.3)

+

Examples.

① Effect of Negative Angle.

$$\sin(-x) = -\sin x$$

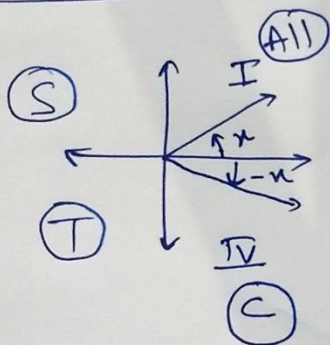
$$\boxed{\cos(-x) = \cos x}$$

$$\tan(-x) = -\tan x$$

$$\operatorname{cosec}(-x) = -\operatorname{cosec} x$$

$$\boxed{\sec(-x) = \sec x}$$

$$\cot(-x) = -\cot x$$



② ★ $\square\square \pm \square\square$

$$\cos(x+y) = \cos x \cdot \cos y - \sin x \cdot \sin y$$

$$\cos(x-y) = \cos x \cdot \cos y + \sin x \cdot \sin y$$

$$\sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y$$

$$\sin(x-y) = \sin x \cdot \cos y - \cos x \cdot \sin y$$

③ Multiply to Add / subtract
(x) (+, -)

$$2 \sin x \cdot \cos y = \sin(x+y) + \sin(x-y)$$

$$2 \cos x \cdot \sin y = \sin(x+y) - \sin(x-y)$$

$$2 \cos x \cdot \cos y = \cos(x+y) + \cos(x-y)$$

$$\star 2 \sin x \cdot \sin y = \cos(x-y) - \cos(x+y)$$

③ $\cot^{-1} \frac{1}{2}$

④ $\oplus$ or $\ominus \longrightarrow \otimes$

Add/Subtract to Multiply.

$$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2 \sin\left(\frac{x-y}{2}\right) \cdot \cos\left(\frac{x+y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)$$

$$\star \cos x - \cos y = 2 \sin\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{y-x}{2}\right)$$

$$= -2 \sin\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)$$

$$\textcircled{5} \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \checkmark$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} \checkmark$$

$$\star \cot(x+y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$$

$$\star \cot(x-y) = \frac{\cot x \cot y + 1}{-\cot x + \cot y}$$

⑥ Multiple Angles.

$$\begin{cases} \sin 2x = 2 \sin x \cdot \cos x \stackrel{\star}{=} \frac{2 \sin x \cdot \cos x}{1} = \frac{(2 \sin x \cos x) / \cos^2 x}{(\sin^2 x + \cos^2 x) / \cos^2 x} = \frac{2 \tan x}{1 + \tan^2 x} \\ \sin 2x = \frac{2 \tan x}{1 + \tan^2 x} \end{cases}$$

$$\begin{cases} \cos 2x = \cos^2 x - \sin^2 x \\ \cos 2x = 2 \cos^2 x - 1 \\ \cos 2x = 1 - 2 \sin^2 x \\ \cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x} \end{cases}$$

$$\begin{cases} \tan 2x = \frac{2 \tan x}{1 - \tan^2 x} \end{cases}$$

$$\begin{aligned} \cos^2 x &= 1 - \sin^2 x \\ \sin^2 x &= 1 - \cos^2 x \end{aligned}$$

$$\sin 3x = \sin(x+2x)$$

$$\sin 3x = 3 \sin x - 4 \sin^3 x \quad \text{Trick} \quad \boxed{31-43}$$

$$\cos 3x = 4 \cos^3 x - 3 \cos x \quad \boxed{43-31}$$

$$\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$$

Application.

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\sin\left(\frac{\pi}{2} + u\right) = \cos u$$

$$\cos\left(\frac{\pi}{2} + u\right) = -\sin u$$

$$\sin(\pi - u) = \sin u$$

$$\cos(\pi - u) = -\cos u$$

$$\sin(\pi + u) = -\sin u$$

$$\cos(\pi + u) = -\cos u$$

$$\sin\left(\frac{3\pi}{2} - u\right) = -\cos u$$

$$\cos\left(\frac{3\pi}{2} - u\right) = -\sin u$$

$$\sin(2\pi - u) = -\sin u$$

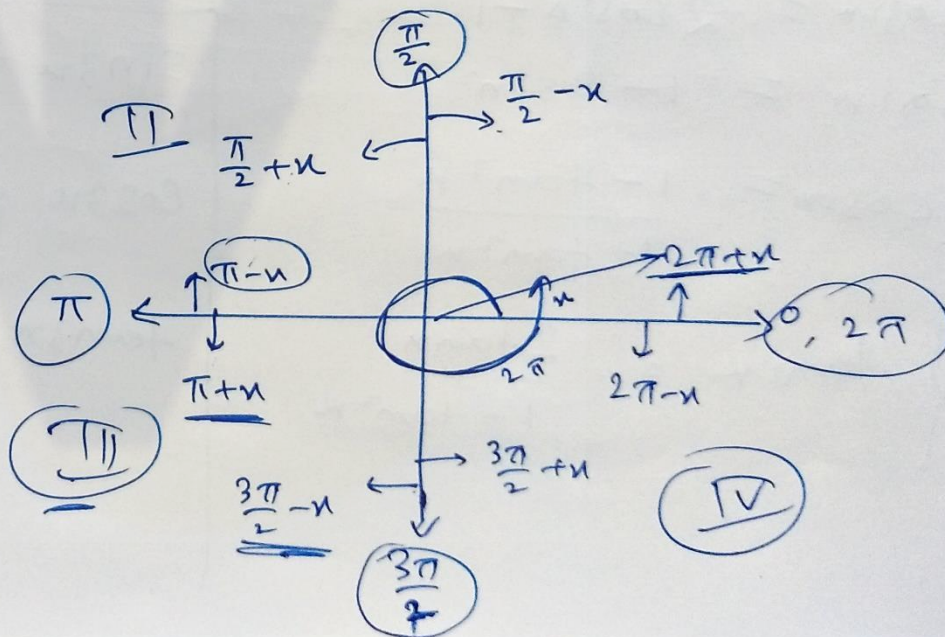
$$\cos(2\pi - u) = +\cos u$$

$$\sin(2\pi + u) = \sin u$$

$$\cos(2\pi + u) = \cos u$$

$$\sin\left(\frac{3\pi}{2} + u\right) = ?$$

$$\cos\left(\frac{3\pi}{2} + u\right) = ?$$



⑤ (i) $\sin 75^\circ$

Example

$$\begin{aligned}
 \textcircled{11} \quad \sin 15^\circ &= \sin (45^\circ - 30^\circ) \\
 &= \sin (45^\circ) \cos (30^\circ) - \cos (45^\circ) \sin (30^\circ) \\
 &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \\
 &= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}
 \end{aligned}$$

① ⑭ ★ Prove

$$\begin{aligned}
 &\tan 3x \cdot \tan 2x \cdot \tan x \\
 &= \tan 3x - \tan 2x - \tan x
 \end{aligned}$$

$$\begin{aligned}
 3x &= (2x + x) \\
 \tan 3x &= \tan (2x + x)
 \end{aligned}$$

$$\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \cdot \tan x}$$

$$\begin{aligned}
 \Rightarrow \tan 3x &= \tan 3x \cdot \tan 2x \cdot \tan x \\
 &= \tan 2x + \tan x
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \tan 3x - \tan 2x - \tan x \\
 &= \tan 3x \cdot \tan 2x \cdot \tan x
 \end{aligned}$$

$$\sec^2 \pi = 2(2)$$

$$10$$

Example, (17)

Prove that

$$\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$$

$$\text{LHS} = \frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x}$$

$$= \frac{2 \sin \left(\frac{6x}{2} \right) \cdot \cos \left(\frac{4x}{2} \right) - 2\sin 3x}{-2 \sin \left(\frac{5x+x}{2} \right) \cdot \sin \left(\frac{5x-x}{2} \right)}$$

$$= \frac{\cancel{2} \sin 3x \cdot \cos 2x - \cancel{2} \sin 3x}{-\cancel{2} \sin 3x \cdot \sin 2x}$$

$$= \frac{\cos 2x - 1}{-\sin 2x} = \frac{1 - \cos 2x}{\sin 2x}$$

$$= \frac{1 - \cos 2x}{\sin 2x}$$

$\begin{aligned} 1 - \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \\ &= \frac{1 - \tan^2 x}{1 + \tan^2 x} \end{aligned}$

$\begin{aligned} \sin 2x &= \frac{2 \sin x \cos x}{2 \tan x} \\ &= \frac{2 \tan x}{1 + \tan^2 x} \end{aligned}$

$$= \frac{1 - (1 - 2\sin^2 x)}{2 \sin x \cos x}$$

$$= \frac{1 - 1 + 2\sin^2 x}{2 \sin x \cdot \cos x}$$

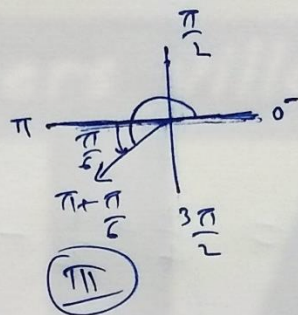
$$= \frac{\sin x}{\cos x}$$

$$= \tan x = \text{RHS.}$$

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Class-11. Maths. Exercise 3.3

$$\operatorname{cosec} \frac{7\pi}{6} = \operatorname{cosec} \left(\overbrace{\pi + \frac{\pi}{6}}^{\text{III}} \right)$$



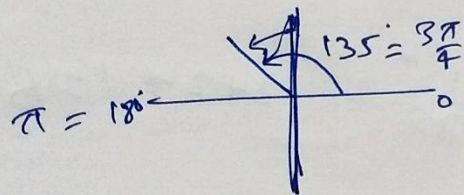
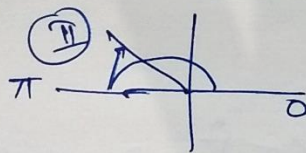
$$= -\operatorname{cosec} \left(\frac{\pi}{6} \right) = -2$$

$$\operatorname{cosec} \frac{5\pi}{6}$$

$$\sin \frac{\pi}{6} = \sin 30^\circ = \frac{1}{2}$$

$$\sin \frac{3\pi}{4}$$

$$\operatorname{cosec} \left(\pi - \frac{\pi}{6} \right) = +\operatorname{cosec} \left(\frac{\pi}{6} \right) = 2$$



$$\sin \left(\frac{3\pi}{4} \right) = \sin \left(\overbrace{\frac{\pi}{2} + \frac{\pi}{4}}^{\text{II}} \right) = +\cos \left(\frac{\pi}{4} \right) = \frac{1}{\sqrt{2}}$$

$$\sin \left(\frac{\pi}{2} + u \right)$$

$$\operatorname{cosec} \frac{7\pi}{6} = -2$$

$$\operatorname{cosec} \frac{5\pi}{6} = +2$$

$$\sin \frac{3\pi}{4} = +\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{6} = 30^\circ$$

$$\frac{\pi}{4} = 45^\circ$$

$$\frac{\pi}{3} = 60^\circ$$

$$\textcircled{1} \quad \sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = \frac{1}{4} + \frac{1}{4} - 1 = \frac{1}{2} - 1 = -\frac{1}{2} \quad \checkmark$$

$$\textcircled{2} \quad 2 \sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cdot \cos^2 \frac{\pi}{3} = 2 \left(\frac{1}{4} \right) + (-2)^2 \cdot \left(\frac{1}{4} \right) = \frac{1}{2} + \cancel{4} \times \frac{1}{4} = \frac{3}{2} \quad \checkmark$$

$$\textcircled{3} \quad \cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3 \tan^2 \frac{\pi}{6} = (\sqrt{3})^2 + 2 + \cancel{3} \left(\frac{1}{3} \right) = 3 + 2 + 1 = 6 \quad \checkmark$$

$$\begin{aligned} \textcircled{4} \quad 2 \sin^2 \left(\frac{3\pi}{4} \right) + 2 \cos^2 \frac{\pi}{4} + 2 \sec^2 \frac{\pi}{3} &= 2 \left(\frac{1}{2} \right) + 2 \left(\frac{1}{2} \right) + 2(4) = \\ &= 1 + 1 + 8 = 10 \quad \checkmark \end{aligned}$$

⑤ (i) $\sin 75^\circ$

$$= \sin(45^\circ + 30^\circ)$$

$$\sin(x+y) = \sin x \cdot \cos y + \sin y \cdot \cos x$$

$\begin{matrix} & \nearrow & & \nearrow \\ 45^\circ & & 30^\circ & & 30^\circ & & 45^\circ \end{matrix}$

$$\sin(75^\circ) = \sin 45^\circ \cdot \cos 30^\circ + \sin 30^\circ \cdot \cos 45^\circ$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}} \quad \checkmark$$

(ii) $\tan 15^\circ$

$$15 = 45 - 30 \quad \checkmark$$

$$15 = 60 - 45 \quad \checkmark$$

$$\tan(15^\circ) = \tan(45^\circ - 30^\circ)$$

$\begin{matrix} (x) & (y) \end{matrix}$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \cdot \tan y}$$

$$x = 45$$

$$y = 30$$

$$\tan 15^\circ = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

Exercise 3.3 Q6 — Q11 (~~use~~ Formulae)

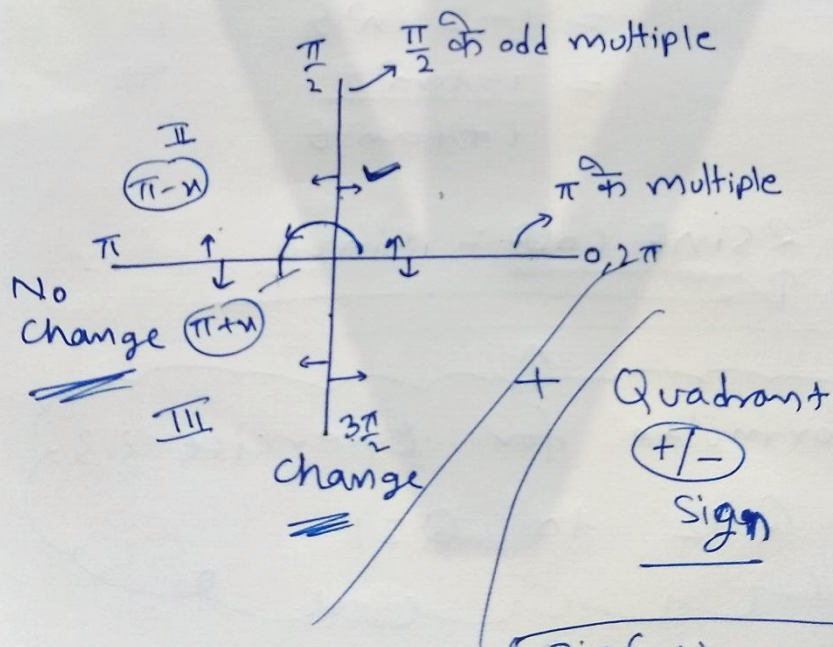
$$\cos A \cos B - \sin A \sin B = \cos(A+B)$$

$$\cos A \cos B + \sin A \sin B = \cos(A-B)$$

$$\sin A \cos B + \cos A \sin B = \sin(A+B)$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$$



$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

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Class 11 Maths Exercise 3.3

$$\textcircled{6} \cos\left(\frac{\pi}{4} - x\right) \cdot \cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right) \sin\left(\frac{\pi}{4} - y\right) \\ = \sin(x+y)$$

$$\text{L.H.S.} = \underbrace{\cos\left(\frac{\pi}{4} - x\right)}_A \cdot \underbrace{\cos\left(\frac{\pi}{4} - y\right)}_B - \underbrace{\sin\left(\frac{\pi}{4} - x\right)}_A \cdot \underbrace{\sin\left(\frac{\pi}{4} - y\right)}_B$$

$$\cos A \cos B - \sin A \sin B = \cos(A+B)$$

$$= \cos\left(\frac{\pi}{4} - x + \frac{\pi}{4} - y\right)$$

$$= \cos\left(\frac{\pi}{2} - \underbrace{(x+y)}_{\theta}\right)$$

$$= \sin(x+y)$$

$$= \text{RHS.}$$

$$\left(\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta\right)$$

⑦

$$\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$$

$$\text{LHS} = \frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)}$$

$$= \frac{\left(\tan \frac{\pi}{4} + \tan x\right)}{1 - \tan \frac{\pi}{4} \cdot \tan x}$$

$$\frac{\left(\tan \frac{\pi}{4} - \tan x\right)}{1 + \tan \frac{\pi}{4} \cdot \tan x}$$

$$\boxed{\tan \frac{\pi}{4} = \tan 45^\circ = 1}$$

$$= \frac{\left(\frac{1 + \tan x}{1 - \tan x}\right)}{\left(\frac{1 - \tan x}{1 + \tan x}\right)} = \frac{(1 + \tan x)^2}{(1 - \tan x)^2} = \text{RHS.}$$

$$\textcircled{8} \quad \frac{\cos(\pi+x) \cdot \cos(-x)}{\sin(\pi-x) \cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$$

$$\text{LHS} = \frac{\cos(\pi+x) \cdot \cos(-x)}{\sin(\pi-x) \cdot \cos\left(\frac{\pi}{2}+x\right)}$$

10th class $\cos(90-\theta) = \sin \theta$
 $\hookrightarrow \cos\left(\frac{\pi}{2}-x\right) = \sin x$

III
 $\cos(\pi+x) = -\cos x$
 $\sin(\pi-x) = +\sin x$
 (IV)
 $\cos\left(\frac{\pi}{2}+x\right) = -\sin x$

$$\text{LHS} = \frac{(-\cos x) \cdot (\cos x)}{(\sin x) \cdot (-\sin x)}$$

$$= \frac{\cos^2 x}{\sin^2 x} = \cot^2 x = \text{RHS}$$

⑨

$$\cos\left(\frac{3\pi}{2}+x\right) \cdot \cos(2\pi+x) \cdot \left[\cot\left(\frac{3\pi}{2}-x\right) + \cot(2\pi+x)\right]$$

$$\cos\left(\frac{3\pi}{2}+x\right) = +\sin x$$

IV

$$\cos(2\pi+x) = +\cos x$$

$$\cot\left(\frac{3\pi}{2}-x\right) = +\tan x$$

III

$$\cot(2\pi+x) = +\cot x$$

$$\text{LHS} = \sin x \cdot \cos x \cdot [\tan x + \cot x]$$

$$= \sin x \cdot \cos x \left[\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right]$$

$$= \sin x \cdot \cos x \left[\frac{\sin^2 x + \cos^2 x}{\cos x \sin x} \right]$$

$$= 1 = \text{RHS} \quad \checkmark$$

$$(10) \sin(n+1)x \cdot \sin(n+2)x + \cos(n+1)x \cdot \cos(n+2)x = \cos x$$

$$\text{LHS.} = \sin \underbrace{(n+1)x}_A \cdot \sin \underbrace{(n+2)x}_B + \cos \underbrace{(n+1)x}_A \cdot \cos \underbrace{(n+2)x}_B$$

$$\boxed{\sin A \cdot \sin B + \cos A \cdot \cos B = \cos(A-B)}$$

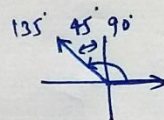
$$= \cos \left(\overset{(n+1)x}{\uparrow} A - \overset{(n+2)x}{\uparrow} B \right)$$

$$= \cos((n+1)x - (n+2)x) = \cos(\cancel{nx} + x - \cancel{nx} - 2x) = \cos(-x) = \cos x = \underline{\underline{\text{RHS}}}$$

$$(11) \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x$$

$$\text{LHS.} = \underbrace{\cos\left(\frac{3\pi}{4} + x\right)}_{\cos(A+B)} - \underbrace{\cos\left(\frac{3\pi}{4} - x\right)}_{\cos(A-B)}$$

$$\begin{aligned} \cos \frac{3\pi}{4} &= -\frac{1}{\sqrt{2}} \\ \sin \frac{3\pi}{4} &= \frac{1}{\sqrt{2}} \end{aligned}$$



$$\boxed{2 = \sqrt{2} \cdot \sqrt{2}}$$

$$= \left[\cancel{\cos \frac{3\pi}{4}} \cdot \cos x - \underbrace{\sin \frac{3\pi}{4}}_{\frac{1}{\sqrt{2}}} \cdot \sin x \right] - \left[\cancel{\cos \frac{3\pi}{4}} \cdot \cos x + \sin \frac{3\pi}{4} \cdot \sin x \right]$$

$$= -\frac{1}{\sqrt{2}} \cdot \sin x - \frac{1}{\sqrt{2}} \cdot \sin x = -2\left(\frac{1}{\sqrt{2}} \cdot \sin x\right) = -\sqrt{2} \sin x = \text{RHS.}$$

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Class - 11 - Maths x Exercise 3.3

(12) $\sin^2 6x - \sin^2 4x = \sin 2x \cdot \sin 10x$

LHS = $\sin^2 6x - \sin^2 4x$ $a^2 - b^2$

= $(\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$

= $\left[2 \sin\left(\frac{6x+4x}{2}\right) \cdot \cos\left(\frac{6x-4x}{2}\right) \right] \cdot \left[2 \sin\left(\frac{6x-4x}{2}\right) \cdot \cos\left(\frac{6x+4x}{2}\right) \right]$

= $4 \cdot \sin 5x \cdot \cos x \cdot \sin x \cdot \cos 5x$

= $(2 \sin 5x \cdot \cos 5x) \cdot (2 \sin x \cdot \cos x)$

= $(\sin 10x) \cdot (\sin 2x)$

= RHS.

Apply $\begin{cases} \sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) \\ \sin A - \sin B = 2 \sin\left(\frac{A-B}{2}\right) \cdot \cos\left(\frac{A+B}{2}\right) \end{cases}$

(Here $\underline{2 \sin \theta \cdot \cos \theta = \sin 2\theta}$)

$$(13) \cos^2 2x - \cos^2 6x = \sin 4x \cdot \sin 8x$$

$$\text{LHS} = \cos^2 2x - \cos^2 6x \quad (a^2 - b^2)$$

$$= (\cos 2x + \cos 6x) (\cos 2x - \cos 6x)$$

$$= \left[2 \cos \left(\frac{2x+6x}{2} \right) \cdot \cos \left(\frac{2x-6x}{2} \right) \right] \cdot \left[-2 \sin \left(\frac{2x+6x}{2} \right) \cdot \sin \left(\frac{2x-6x}{2} \right) \right]$$

$$= -4 \cos(4x) \cdot \cos(-2x) \cdot \sin(4x) \cdot \sin(-2x)$$

$$\left\{ \begin{array}{l} \cos(-\theta) = \cos \theta \\ \sin(-\theta) = -\sin \theta \end{array} \right\}$$

$$= 4 \cos 4x \cdot \cos 2x \cdot \sin 4x \cdot \sin 2x$$

$$= (2 \cos 4x \cdot \sin 4x) \cdot (2 \cos 2x \cdot \sin 2x)$$

$$= \sin 8x \cdot \sin 4x$$

$$= \text{RHS.}$$

$$\begin{aligned} \text{Apply } \cos A + \cos B &= 2 \cos \frac{A+B}{2} \cdot \cos \frac{A-B}{2} \\ \cos A - \cos B &= -2 \sin \frac{A+B}{2} \cdot \sin \frac{A-B}{2} \end{aligned}$$

$$(14) \quad \sin 2x + 2\sin 4x + \sin 6x = \underline{4\cos^2 x \cdot \sin 4x}$$

$$\text{LHS} = \sin 2x + 2\sin 4x + \sin 6x$$

$$= (\sin 2x + \sin 6x) + 2\sin 4x$$

$$\boxed{\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)}$$

$$= 2\sin(4x) \cdot \cos(-2x) \quad \left(\cos(-\theta) = \cos\theta \right)$$

$$= \underline{2\sin 4x \cdot \cos 2x} + \underline{2\sin 4x}$$

$$= \underline{2\sin 4x} \cdot \{ \underline{\cos 2x} + 1 \}$$

$$\left(\because \cos 2\theta = 2\cos^2\theta - 1 \right)$$

$$= 2\sin 4x \cdot \{ \underline{2\cos^2 x - 1} + 1 \}$$

$$= 4\cos^2 x \cdot \sin 4x = \text{RHS.} \quad \checkmark$$

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Class - 11 - Maths x Exercise 3.3

$$(15) \quad \cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$$

$$\Rightarrow \frac{\cos 4x}{\sin 4x} \left(2 \sin \left(\frac{5x+3x}{2} \right) \cdot \cos \left(\frac{5x-3x}{2} \right) \right) = \frac{\cos x}{\sin x} \left(2 \sin \left(\frac{5x-3x}{2} \right) \cdot \cos \left(\frac{5x+3x}{2} \right) \right)$$

$$\Rightarrow \frac{\cos 4x}{\cancel{\sin 4x}} (2 \cdot \cancel{\sin 4x} \cdot \cos x) = \frac{\cancel{\cos x}}{\cancel{\sin x}} (2 \cancel{\sin x} \cdot \cos 4x)$$

$$\Rightarrow 2 \cdot \cos x \cdot \cos 4x = 2 \cos x \cdot \cos 4x$$

$$\boxed{LHS = RHS} \quad \checkmark$$

$$(16) \quad \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = - \frac{\sin 2x}{\cos 10x} \quad \rightarrow \quad = \frac{-2 \cancel{\sin (7x)} \cdot \sin (2x)}{2 \cancel{\sin (7x)} \cdot \cos (10x)}$$

$$LHS = \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$$

$$= - \frac{\sin 2x}{\cos 10x} = \underline{RHS.}$$

$$\left\{ \begin{array}{l} \text{APPLY} \\ \cos A - \cos B = \\ \sin A - \sin B = \end{array} \right.$$

$$(17) \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$$

$$\text{LHS} = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} \quad \left(\begin{array}{l} \text{Apply} \\ \sin A + \sin B = \dots \\ \cos A + \cos B = \dots \end{array} \right)$$

$$= \frac{2 \sin \left(\frac{5x+3x}{2} \right) \cdot \cos \left(\frac{5x-3x}{2} \right)}{2 \cos \left(\frac{5x+3x}{2} \right) \cdot \cos \left(\frac{5x-3x}{2} \right)}$$

$$= \frac{\cancel{2} \sin 4x \cdot \cancel{\cos x}}{\cancel{2} \cos 4x \cdot \cancel{\cos x}}$$

$$= \tan 4x$$

$$(18) \frac{\sin x - \sin y}{\cos x + \cos y} = \tan \left(\frac{x-y}{2} \right)$$

$$\text{LHS} = \frac{\sin x - \sin y}{\cos x + \cos y} \quad \left(\begin{array}{l} \text{Apply} \\ \sin A - \sin B = \dots \\ \cos A + \cos B = \dots \end{array} \right)$$

$$= \frac{\cancel{2} \sin \left(\frac{x-y}{2} \right) \cdot \cos \left(\frac{x+y}{2} \right)}{\cancel{2} \cos \left(\frac{x+y}{2} \right) \cdot \cos \left(\frac{x-y}{2} \right)}$$

$$= \frac{\sin \left(\frac{x-y}{2} \right)}{\cos \left(\frac{x-y}{2} \right)} = \tan \left(\frac{x-y}{2} \right) = \underline{\text{RHS}}$$

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Class 11 Maths × Exercise 3.3

$$(19) \quad \frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$$

$$\text{LHS} = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$$

$$= \frac{\cancel{2} \sin\left(\frac{x+3x}{2}\right) \cdot \cancel{\cos\left(\frac{x-3x}{2}\right)}}{\cancel{2} \cos\left(\frac{x+3x}{2}\right) \cdot \cancel{\cos\left(\frac{x-3x}{2}\right)}}$$

$$= \tan 2x = \underline{\text{RHS.}}$$

(20)

$$\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2 \sin x$$

$$\text{LHS} = \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} \rightarrow \begin{array}{l} \text{Sin A - Sin B} \\ \text{Cos 2x = cos}^2 x - \text{sin}^2 x \\ \text{① } \underline{-\cos 2x = -\cos^2 x + \sin^2 x} \end{array}$$

$$= \frac{2 \sin\left(\frac{A-B}{2}\right) \cdot \cos\left(\frac{A+B}{2}\right)}{-\cos 2x}$$

$$= \frac{2 \sin\left(\frac{x-3x}{2}\right) \cdot \cos\left(\frac{x+3x}{2}\right)}{-\cos 2x}$$

$$= \frac{2 \sin(-x) \cdot \cancel{\cos 2x}}{-\cos 2x} = \frac{-2 \sin x}{-1}$$

$$= 2 \sin x = \text{RHS.}$$

21) ★

$$\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$$

$$\text{LHS} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2 \cos(3x) \cdot \cos x + \cos 3x}{2 \sin 3x \cdot \cos x + \sin 3x}$$

$$= \frac{\cos 3x [2 \cancel{\cos x} + 1]}{\sin 3x [2 \cancel{\cos x} + 1]} = \cot 3x = \text{RHS. } \checkmark$$

$$\begin{aligned} \cos(A) + \cos B &= 2 \cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) \\ &= 2 \cos \frac{7x}{2} \cdot \cos \frac{x}{2} + \cos 2x \end{aligned}$$

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$$

Question 3

$$a^2 - b^2 = (a+b)(a-b)$$

$$\begin{cases} \sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) \\ \sin A - \sin B = 2 \sin\left(\frac{A-B}{2}\right) \cdot \cos\left(\frac{A+B}{2}\right) \\ \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) \\ \star \cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right) \end{cases}$$

$$\begin{aligned} \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= 2\cos^2 \theta - 1 \quad \checkmark \\ &= 1 - 2\sin^2 \theta \\ &= \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \end{aligned}$$

$$\uparrow \underline{2 \sin \theta \cdot \cos \theta = \sin 2\theta}$$

Formulae for Exercise 3.3.

Q 12 to Q 21

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Class-11-Maths Exercise 3.3

Q.22

$$\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$$

~~$\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$~~

Proof:

$$3x = (2x + x)$$

By taking 'tan' both sides

$$\Rightarrow \tan 3x = \tan(2x + x)$$

$$\Rightarrow \tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \cdot \tan x}$$

$$\tan \rightarrow \frac{1}{\cot}$$

$$\Rightarrow \frac{1}{\cot 3x} = \frac{\frac{1}{\cot 2x} + \frac{1}{\cot x}}{\frac{1}{1} - \frac{1}{\cot 2x} \cdot \frac{1}{\cot x}}$$

$$\Rightarrow \frac{1}{\cot 3x} = \frac{\frac{\cot x + \cot 2x}{\cot x \cdot \cot 2x}}{\frac{\cot x \cdot \cot 2x - 1}{\cot x \cdot \cot 2x}}$$

$$\Rightarrow \frac{1}{\cot 3x} = \frac{\cot x + \cot 2x}{\cot x \cdot \cot 2x - 1}$$

$$\Rightarrow \cot x \cdot \cot 2x - 1 = \cot 3x \cdot \cot x + \cot 3x \cdot \cot 2x$$

$$\Rightarrow \cot x \cdot \cot 2x - \cot 3x \cdot \cot x - \cot 3x \cdot \cot 2x = 1$$

H.P.

Q.23

$$\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$$

LHS. = $\tan 4x$

$$= \tan(2(2x))$$

$$= \frac{2 \tan(2x)}{1 - \tan^2(2x)}$$

$$= \frac{2 \left(\frac{2 \tan x}{1 - \tan^2 x} \right)}{\frac{1}{1} - \left(\frac{2 \tan x}{1 - \tan^2 x} \right)^2}$$

$$= \frac{\frac{4 \tan x}{1 - \tan^2 x}}{\frac{(1 - \tan^2 x)^2 - (2 \tan x)^2}{(1 - \tan^2 x)^2}}$$

$\tan 2\theta$
 $\Downarrow$
 $\frac{2 \tan \theta}{1 - \tan^2 \theta}$

$\tan 2x$
 $= \frac{2 \tan x}{1 - \tan^2 x}$

$\frac{2x}{2x}$
 $\frac{2x}{2x}$
 $\frac{2x}{2x}$
 $\frac{2x}{2x}$

$$= \frac{4 \tan x (1 - \tan^2 x)}{1 + \tan^4 x - 2 \tan^2 x - 4 \tan^2 x}$$

$$= \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$$

$$= \text{RHS.}$$

Q 22, Q 23

$$\times \cot(x+y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$$

$$\checkmark \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$$

$$\checkmark \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

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Class-11- maths. Exercise 3.3

Q.24

Prove that

$$\cos 4x = 1 - 8 \sin^2 x \cdot \cos^2 x$$

Proof:

$$\begin{aligned} \text{LHS} &= \cos 4x \\ &= 1 - 2 \sin^2(2x) \quad \left(\because \cos 2\theta = 1 - 2 \sin^2 \theta \right) \\ &= 1 - 2 (\sin 2x)^2 \quad \left(\theta = 2x \right) \\ &= 1 - 2 (2 \sin x \cdot \cos x)^2 \quad \left(\sin 2x = 2 \sin x \cdot \cos x \right) \\ &= 1 - 2 (4 \sin^2 x \cdot \cos^2 x) \\ &= 1 - 8 \sin^2 x \cdot \cos^2 x \\ &= \text{RHS.} \end{aligned}$$

$$\cos 2x = \begin{cases} \cos^2 x - \sin^2 x \\ 2 \cos^2 x - 1 \\ 1 - 2 \sin^2 x \\ \frac{1 - \tan^2 x}{1 + \tan^2 x} \end{cases}$$

$$\sin 2x = \begin{cases} 2 \sin x \cos x \\ \frac{2 \tan x}{1 + \tan^2 x} \end{cases}$$

$$\cos 3x = 4 \cos^3 x - 3 \cos x$$

Q.25

Prove that

$$\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$$

$$\text{LHS} = \cos 6x$$

$$= \cos 3(2x)$$

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$= 4 \cos^3(2x) - 3 \cos(2x)$$

$$= 4 (\cos 2x)^3 - 3 (\cos 2x)$$

$$= 4 (2 \cos^2 x - 1)^3 - 3 (2 \cos^2 x - 1)$$

$$\Rightarrow (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$= 4 [(2 \cos^2 x)^3 - 3 \cdot (2 \cos^2 x)^2 \cdot 1$$

$$+ 3 (2 \cos^2 x) \cdot 1^2 - 1^3]$$

$$- 6 \cos^2 x + 3$$

$$= 4 (8 \cos^6 x - 12 \cos^4 x + 6 \cos^2 x - 1)$$

$$- 6 \cos^2 x + 3$$

$$= 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$$

Q24

Q25

Toppers Village.

Trigonometric Equations.

Before exercise 3.4

Algebraic equations

No. of Solutions → Fixed.

$$2x-1=0 \rightarrow x=\frac{1}{2}$$

$$x^2-5x+6=0$$

$$\rightarrow x=2,3$$

Trigonometric equations.

No. of Solutions → ∞

Unknown
↓
Angle

$$\sin x = \frac{1}{2}$$

Angle

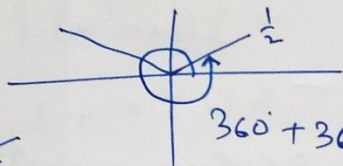
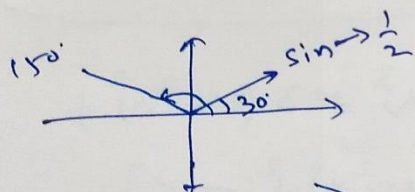
$$\theta = 30^\circ = \frac{\pi}{6}$$

$$\theta = 150^\circ = \frac{5\pi}{6}$$

$$\theta = 390^\circ = \frac{7\pi}{6}$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\sin 150^\circ = \frac{1}{2}$$



$$360^\circ + 30^\circ = 390^\circ$$

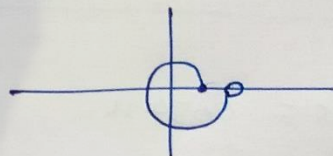
$$\theta = 750^\circ$$



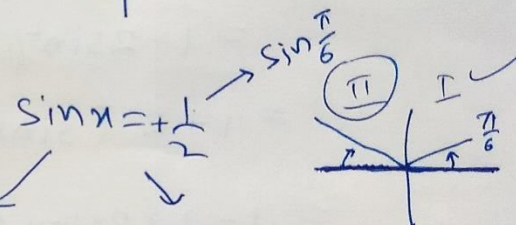
Principal solution.

Angle $\rightarrow [0, 2\pi)$

$$0 \leq \text{Angle} < 2\pi$$



e.g.



$$x = \frac{\pi}{6}$$

$$x = \frac{5\pi}{6} = \pi - \frac{\pi}{6}$$

(I)

(II)

$$\text{Pri. Sol.} = \frac{\pi}{6}, \frac{5\pi}{6} \in [0, 2\pi)$$

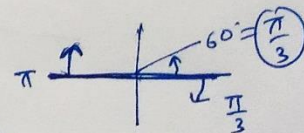
$$\tan x = -\sqrt{3}$$

II

$$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

IV

$$2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$



General Solutions of Trigonometric Equations.

$\sin x = \sin \theta$	$x = n\pi + (-1)^n \theta$
$\csc x = \csc \theta$	$x = n\pi + (-1)^n \theta$
$\cos x = \cos \theta$	$x = 2n\pi \pm \theta$
$\sec x = \sec \theta$	$x = 2n\pi \pm \theta$
$\tan x = \tan \theta$	$x = n\pi + \theta$
$\cot x = \cot \theta$	$x = n\pi + \theta$

$n \in \mathbb{Z}$

Proof: (I) $\sin x = \sin \theta$

$$\Rightarrow \sin x - \sin \theta = 0$$

$$\Rightarrow 2 \sin\left(\frac{x-\theta}{2}\right) \cdot \cos\left(\frac{x+\theta}{2}\right) = 0$$

$$\sin\left(\frac{x-\theta}{2}\right) = 0$$

$$\Rightarrow \frac{x-\theta}{2} = n\pi$$

$$\Rightarrow \boxed{x = 2n\pi + \theta}$$

Even $\oplus$

$$\cos\left(\frac{x+\theta}{2}\right) = 0$$

$$\frac{x+\theta}{2} = (2n+1)\frac{\pi}{2}$$

$$\boxed{x = (2n+1)\pi - \theta}$$

odd $\ominus$

$$\begin{matrix} (n-1)(n-2)=0 \\ \downarrow \\ x=1 \quad x=2 \end{matrix}$$

$$x = 2n\pi + \theta$$

$$x = (2n+1)\pi - \theta$$

$$\boxed{x = n\pi + (-1)^n \theta}$$

$n = \text{even}$
 $n = \text{odd}$

(II) $\cos x = \cos \theta$

$$\Rightarrow \cos x - \cos \theta = 0$$

$$\Rightarrow -2 \sin\left(\frac{x+\theta}{2}\right) \cdot \sin\left(\frac{x-\theta}{2}\right) = 0$$

$$\sin\left(\frac{x+\theta}{2}\right) = 0$$

$$\frac{x+\theta}{2} = n\pi$$

$$\boxed{x = 2n\pi - \theta}$$

$$\boxed{x = 2n\pi \pm \theta}$$

$$\sin\left(\frac{x-\theta}{2}\right) = 0$$

$$\frac{x-\theta}{2} = n\pi$$

$$\boxed{x = 2n\pi + \theta}$$

$$\textcircled{\text{III}} \quad \tan u = \tan \theta$$

$$\Rightarrow \tan u - \tan \theta = 0$$

$$\Rightarrow \frac{\sin u}{\cos u} - \frac{\sin \theta}{\cos \theta} = 0$$

$$\Rightarrow \frac{\sin u \cdot \cos \theta - \cos u \cdot \sin \theta}{\cos u \cdot \cos \theta} = 0$$

$$\Rightarrow \sin u \cdot \cos \theta - \cos u \cdot \sin \theta = 0$$

$$\Rightarrow \sin(u - \theta) = 0$$

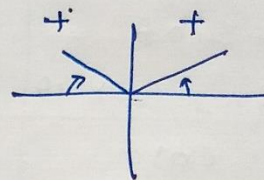
$$\Rightarrow u - \theta = n\pi$$

$$\boxed{u = n\pi + \theta}$$

Note.

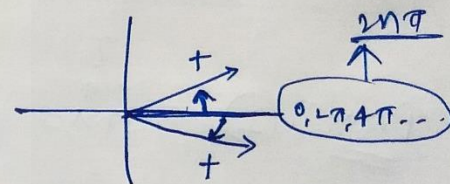
$$\sin u = \sin \theta$$

$$\boxed{u = n\pi + (-1)^n \theta}$$



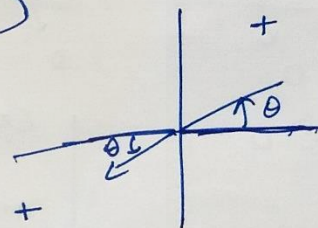
$$\cos u = \cos \theta$$

$$\boxed{u = 2n\pi \pm \theta}$$



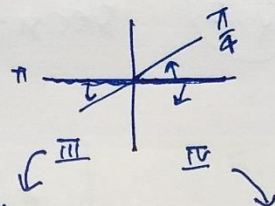
$$\tan u = \tan \theta$$

$$\boxed{u = n\pi + \theta}$$



Example

General Solⁿ:



$$\sin x = -\frac{1}{\sqrt{2}} = \sin\left(\pi + \frac{\pi}{4}\right) \quad \text{or} \quad \sin\left(2\pi - \frac{\pi}{4}\right)$$

$$\sin x = \sin\left(\frac{5\pi}{4}\right)$$

$$x = n\pi + (-1)^n \theta \quad n \in \mathbb{I}$$

General Solⁿ:

$$x = n\pi + (-1)^n \cdot \frac{5\pi}{4}$$

Formula:

$$n = 0$$
$$x = \frac{5\pi}{4} \quad \checkmark$$

$$\sin x = \sin \frac{7\pi}{4}$$

$$x = n\pi + (-1)^n \cdot \frac{7\pi}{4} \quad n \in \mathbb{I}$$

Both are correct.

But look different

$$n = 3$$

$$x = 3\pi - \frac{7\pi}{4} = \frac{12\pi - 7\pi}{4} = \frac{5\pi}{4}$$

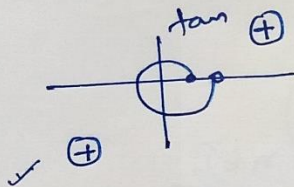
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Class - 11 - Maths x Exercise 3.4

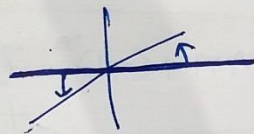
Principal & General Solutions

Trigonometric Equations

Q.1 $\tan x = \sqrt{3}$



$x = \frac{\pi}{3} = 60^\circ$



Principal solⁿ $[0, 2\pi)$

$$\left. \begin{array}{l} \text{I} \rightarrow \frac{\pi}{3} = \frac{\pi}{3} \\ \text{III} \rightarrow \pi + \frac{\pi}{3} = \frac{4\pi}{3} \end{array} \right\}$$

General solⁿ

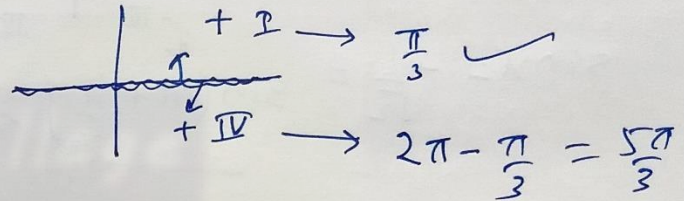
$$\sqrt{3} = \tan x = \tan \frac{\pi}{3}$$

$x = n\pi + \frac{\pi}{3}$

Q.2

$$\sec x = 2 = \sec \frac{\pi}{3}$$

$\cos x = \frac{1}{2} \rightarrow x = \frac{\pi}{3}$



Principal solⁿ $= \frac{\pi}{3}, \frac{5\pi}{3}$

General solⁿ

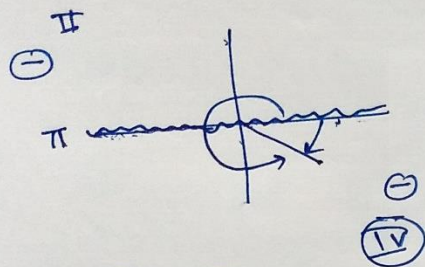
$$x = 2n\pi \pm \theta, \quad n \in \mathbb{Z}$$

$$x = 2n\pi \pm \frac{\pi}{3}$$

Q.3

$$\cot \theta = -\sqrt{3} = \cot \frac{5\pi}{6} = \cot \frac{11\pi}{6}$$

$$\left(30^\circ, \frac{\pi}{6} \right)$$



Principal, $\Rightarrow \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\Rightarrow 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

General solⁿ,

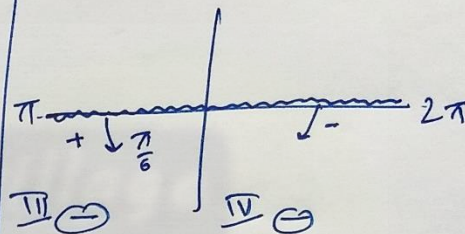
$$\theta = n\pi + \theta, n \in \mathbb{Z}$$

$$\theta = n\pi + \frac{5\pi}{6}$$

Q.4

$$\cos \theta = -2$$

$$\left(\frac{\pi}{6}, 30^\circ \right)$$



Principal solⁿ $\Rightarrow \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\Rightarrow 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

General,

$$\theta = n\pi + (-1)^n \cdot \theta$$

$$\theta = n\pi + (-1)^n \frac{7\pi}{6}$$

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Class - II - Maths x Exercise 3.4

Q.5

General solutions

$$\cos 4x = \cos 2x$$

$$\Rightarrow 4x = 2n\pi \pm 2x$$

$$\begin{array}{c} \cos x = \cos \theta \\ \downarrow \\ x = 2n\pi \pm \theta \end{array}$$

$$n \in \mathbb{I}$$

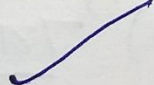
⊕

$$4x = 2n\pi + 2x$$

$$\Rightarrow 4x - 2x = 2n\pi$$

$$\Rightarrow 2x = 2n\pi$$

$$\boxed{x = n\pi}, n \in \mathbb{I}$$



⊖

$$4x = 2n\pi - 2x$$

$$\Rightarrow 4x + 2x = 2n\pi$$

$$\Rightarrow 6x = 2n\pi$$

$$\Rightarrow \boxed{x = \frac{n\pi}{3}}, n \in \mathbb{I}$$



$$\textcircled{6} \quad \cos 3x + \cos x - \cos 2x = 0$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cdot \cos \frac{A-B}{2}$$

$$\Rightarrow 2 \cos \frac{3x+x}{2} \cdot \cos \frac{3x-x}{2} - \cos 2x = 0$$

$$\Rightarrow 2 \cos 2x \cdot \cos x - \cos 2x = 0$$

$$\Rightarrow \cos 2x \cdot [2 \cos x - 1] = 0$$

$$\cos 2x = 0$$

$$\Rightarrow 2x = (2n+1) \frac{\pi}{2}$$

$$\Rightarrow \boxed{x = (2n+1) \frac{\pi}{2}}$$

$$2 \cos x - 1 = 0$$

$$\cos x = \frac{1}{2} = \cos \frac{\pi}{3}$$

General solⁿ

$$\Rightarrow \boxed{x = 2n\pi \pm \frac{\pi}{3}}$$

$$\textcircled{7} \quad \sin 2x + \cos x = 0$$

$$\Rightarrow 2 \sin x \cdot \cos x + \cos x = 0$$

$$\Rightarrow \underbrace{\cos x}_0 \cdot \underbrace{[2 \sin x + 1]}_0 = 0$$

$$\cos x = 0$$

$$\boxed{x = (2n+1) \frac{\pi}{2}}$$

$$2 \sin x + 1 = 0$$

$$\sin x = -\frac{1}{2}$$

$$\sin x = \sin \left(\pi + \frac{\pi}{6} \right)$$

$$\sin x = \sin \frac{7\pi}{6}$$

$$\Rightarrow \boxed{x = n\pi + (-1)^n \frac{7\pi}{6}}$$

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class - 11 - Maths x Exercise 3.4

Q.8 $\sec^2 2x = 1 + \tan^2 2x$

$\sec^2 \theta = 1 + \tan^2 \theta$

$\Rightarrow 1 + \tan^2 2x = 1 + \tan^2 2x$

$\Rightarrow \tan^2 2x + \tan^2 2x = 0$

$\Rightarrow \tan 2x \cdot [\tan 2x + 1] = 0$

$\tan 2x = 0$

$\Rightarrow 2x = n\pi$

$\Rightarrow x = \frac{n\pi}{2}$

$\tan 2x = -1$

$\ominus$	$+$
$+$	$-$

$\tan 2x = \tan(\pi - \frac{\pi}{4})$

$\tan 2x = \tan(\frac{3\pi}{4})$

$2x = n\pi + \frac{3\pi}{4}$

$x = \frac{n\pi}{2} + \frac{3\pi}{8}$

Q $\sin x + \sin 3x + \sin 5x = 0$

$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cdot \cos \frac{A-B}{2}$

$\Rightarrow \sin x + \sin 5x + \sin 3x = 0$

$\Rightarrow 2 \sin \frac{6x}{2} \cdot \cos \frac{4x}{2} + \sin 3x = 0$

$\Rightarrow 2 \sin 3x \cdot \cos 2x + \sin 3x = 0$

$\Rightarrow \sin 3x \cdot [2 \cos 2x + 1] = 0$

$3x = n\pi$

$x = \frac{n\pi}{3}$

$2 \cos 2x + 1 = 0$

$\Rightarrow \cos 2x = -\frac{1}{2} = \cos \frac{2\pi}{3}$

$2x = 2n\pi \pm \frac{2\pi}{3}$

$x = n\pi \pm \frac{\pi}{3}$

Miscellaneous Exercise on Chapter 3

Trigonometric Functions

$$\textcircled{1} \quad 2 \cos \frac{\pi}{13} \cdot \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} = 0$$

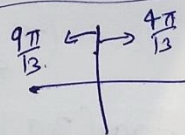
Prove

LHS =

$$2 \cos \frac{\pi}{13} \cdot \cos \frac{9\pi}{13} + 2 \cos \left(\frac{\frac{3\pi}{13} + \frac{5\pi}{13}}{2} \right) \cdot \cos \left(\frac{\frac{5\pi}{13} - \frac{3\pi}{13}}{2} \right)$$

$$= 2 \cos \frac{\pi}{13} \cdot \cos \frac{9\pi}{13} + 2 \cos \left(\frac{4\pi}{13} \right) \cdot \cos \frac{\pi}{13}$$

$$= 2 \cos \frac{\pi}{13} \cdot \left[\cos \frac{9\pi}{13} + \cos \frac{4\pi}{13} \right]$$



$$\frac{9\pi}{13} + \frac{4\pi}{13} = \pi$$

$$\left(\frac{9\pi}{13} \right) = \left(\pi - \frac{4\pi}{13} \right) \Rightarrow \cos \left(\frac{9\pi}{13} \right) = \cos \left(\pi - \frac{4\pi}{13} \right)$$

$$\Rightarrow \cos \frac{9\pi}{13} = -\cos \frac{4\pi}{13}$$

$$= 2 \cos \frac{\pi}{13} \cdot \left\{ -\cos \frac{4\pi}{13} + \cos \frac{4\pi}{13} \right\}$$

$$= 0 = \text{RHS.}$$

$$\begin{cases} \cos A + \cos B = 2 \cos \frac{A+B}{2} \cdot \cos \frac{A-B}{2} \\ \sin A + \sin B = 2 \sin \frac{A+B}{2} \cdot \cos \frac{A-B}{2} \\ \cos A - \cos B = -2 \sin \frac{A+B}{2} \cdot \sin \frac{A-B}{2} \\ \sin A - \sin B = 2 \sin \frac{A-B}{2} \cdot \cos \frac{A+B}{2} \end{cases}$$

+

OBSERVA

$$(2) (\sin 3x + \sin x) \cdot \sin x + (\cos 3x - \cos x) \cdot \cos x = 0$$

Prove

$$\text{LHS} = (\sin 3x + \sin x) \cdot \sin x + (\cos 3x - \cos x) \cdot \cos x$$

$$= \left(2 \sin \frac{3x+x}{2} \cdot \cos \frac{3x-x}{2} \right) \sin x + \left(-2 \sin \frac{3x+x}{2} \cdot \sin \frac{3x-x}{2} \right) \cdot \cos x$$

$$= \cancel{2 \sin 2x \cdot \cos x \cdot \sin x} - \cancel{2 \sin 2x \cdot \sin x \cdot \cos x}$$

$$= 0$$

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Class-11-maths

Misc. Ex. 3

Q. 3

$$(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4 \cos^2 \left(\frac{x+y}{2} \right)$$

↓

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cdot \cos \frac{A-B}{2} \quad \checkmark$$

$$\sin A - \sin B = 2 \sin \frac{A-B}{2} \cdot \cos \frac{A+B}{2} \quad \checkmark$$

$$\text{LHS} = (\cos x + \cos y)^2 + (\sin x - \sin y)^2$$

$$= \left(2 \cos \frac{x+y}{2} \cdot \cos \frac{x-y}{2} \right)^2 + \left(2 \sin \frac{x-y}{2} \cdot \cos \frac{x+y}{2} \right)^2$$

$$= \underline{4 \cos^2 \left(\frac{x+y}{2} \right) \cdot \cos^2 \left(\frac{x-y}{2} \right)} + \underline{4 \sin^2 \left(\frac{x-y}{2} \right) \cdot \cos^2 \left(\frac{x+y}{2} \right)}$$

$$= 4 \cos^2 \left(\frac{x+y}{2} \right) \cdot \left\{ \underbrace{\cos^2 \left(\frac{x-y}{2} \right) + \sin^2 \left(\frac{x-y}{2} \right)}_1 \right\}$$

$$= 4 \cos^2 \left(\frac{x+y}{2} \right) \quad |$$

= RHS.

$$\boxed{Q.4} \quad (\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4 \sin^2 \left(\frac{x-y}{2} \right)$$

$$\text{LHS} = (\cos x - \cos y)^2 + (\sin x - \sin y)^2$$

$$\cos A - \cos B = \underline{-2 \sin \frac{A+B}{2} \cdot \sin \frac{A-B}{2}}, \quad \sin A - \sin B = \underline{2 \sin \frac{A-B}{2} \cdot \cos \frac{A+B}{2}}$$

$$= \left(-2 \sin \frac{x+y}{2} \cdot \sin \frac{x-y}{2} \right)^2 + \left(2 \sin \frac{x-y}{2} \cdot \cos \frac{x+y}{2} \right)^2$$

$$= \underline{4 \sin^2 \left(\frac{x+y}{2} \right) \cdot \sin^2 \left(\frac{x-y}{2} \right)} + \underline{4 \cdot \sin^2 \left(\frac{x-y}{2} \right) \cdot \cos^2 \left(\frac{x+y}{2} \right)}$$

$$= 4 \sin^2 \left(\frac{x-y}{2} \right) \cdot \left\{ \sin^2 \left(\frac{x+y}{2} \right) + \cos^2 \left(\frac{x+y}{2} \right) \right\}$$

$$\Rightarrow 4 \sin^2 \left(\frac{x-y}{2} \right) = \text{RHS.} \quad \rightarrow 1$$

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Miscellaneous Exercise on Chapter 3

Q.5

TO Prove

$$\begin{aligned} & \sin x + \sin 3x + \sin 5x + \sin 7x \\ &= 4 \cos x \cdot \cos 2x \cdot \sin 4x \end{aligned}$$

$$\begin{aligned} \text{LHS} &= \sin x + \sin 3x + \sin 5x + \sin 7x \\ &= 2 \sin \frac{4x}{2} \cdot \cos \frac{2x}{2} + 2 \sin \frac{12x}{2} \cdot \cos \frac{2x}{2} \\ &= 2 \cos x \cdot \{ \sin 2x + \sin 6x \} \\ &= 2 \cos x \cdot \{ 2 \sin 4x \cdot \cos 2x \} \\ &= \text{RHS} \end{aligned}$$

Q.6

$$\text{RHS} = \tan 6x$$

$$\begin{aligned} \text{LHS} &= \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} \\ &= \frac{(\cancel{2} \sin 6x \cdot \cos x) + (\cancel{2} \sin 6x \cdot \cos 3x)}{(\cancel{2} \cos 6x \cdot \cos x) + (\cancel{2} \cos 6x \cdot \cos 3x)} \\ &= \frac{\sin 6x \cdot (\cancel{\cos x} + \cancel{\cos 3x})}{\cos 6x \cdot (\cancel{\cos x} + \cancel{\cos 3x})} \\ &= \tan 6x = \text{RHS} \end{aligned}$$

Q.7

$$\sin 3x + \sin 2x - \sin x = 4 \sin x \cdot \cos \frac{x}{2} \cdot \cos \frac{3x}{2}$$

$$\text{LHS} = \sin 3x + \sin 2x - \sin x$$

$$= (\sin 3x - \sin x) + \sin 2x$$

$$= 2 \sin x \cdot \cos 2x + \sin 2x$$

$$\sin 2x = 2 \sin x \cdot \cos x$$

$$= 2 \sin x \cdot \cos 2x + 2 \sin x \cdot \cos x$$

$$= 2 \sin x \cdot \{ \cos 2x + \cos x \}$$

$$= 2 \sin x \cdot \left\{ 2 \cos \frac{2x+x}{2} \cdot \cos \frac{2x-x}{2} \right\}$$

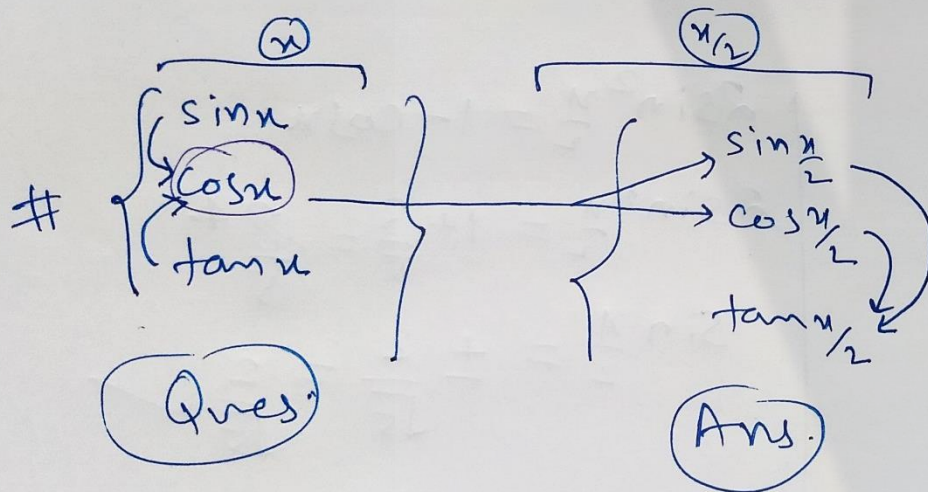
$$= \cancel{2 \sin x} \cdot \{ = 4 \left\{ \sin x \cdot \cos \frac{3x}{2} \cdot \cos \frac{x}{2} \right\} = \text{RHS.}$$

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Miscellaneous Exercise on Chapter 3

Q8, Q9, Q10

$$\# \begin{cases} \cos 2\theta = 2\cos^2 \theta - 1 \rightarrow \cos x = 2\cos^2 \frac{x}{2} - 1 \rightarrow \boxed{2\cos^2 \frac{x}{2} = \cos x + 1} \\ \cos 2\theta = 1 - 2\sin^2 \theta \rightarrow \cos x = 1 - 2\sin^2 \frac{x}{2} \rightarrow \boxed{2\sin^2 \frac{x}{2} = 1 - \cos x} \end{cases}$$



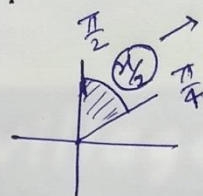
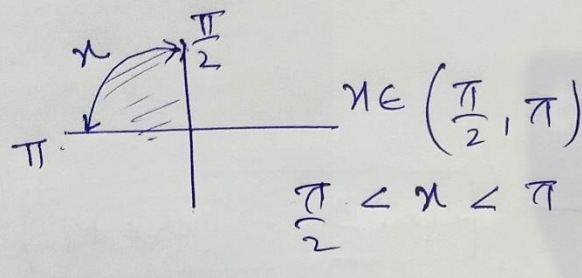
Q.8

$$\tan x = -\frac{4}{3}$$

II quadrant

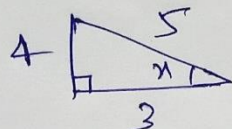
$$\left. \begin{array}{l} \sin \frac{x}{2} ? \\ \cos \frac{x}{2} ? \\ \tan \frac{x}{2} ? \end{array} \right\} \rightarrow$$

I quadrant



$$\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

$$\tan x = -\frac{4}{3} = \frac{P}{B}$$



$$\cos x = \frac{B}{H} = -\frac{3}{5}$$

II

$$2\cos^2 \frac{x}{2} = 1 + \cos x$$

$$\Rightarrow 2\left(\cos \frac{x}{2}\right)^2 = 1 + \left(-\frac{3}{5}\right)$$

$$\Rightarrow \cancel{2} \left(\cos \frac{x}{2}\right)^2 = \frac{2}{5} \Rightarrow \cos \frac{x}{2} = \pm \frac{1}{\sqrt{5}} = +\frac{1}{\sqrt{5}}$$

$$2\sin^2 \frac{x}{2} = 1 - \cos x$$

$$\cancel{2} \sin^2 \frac{x}{2} = 1 + \frac{3}{5} = \frac{8}{5}$$

$$\sin \frac{x}{2} = \pm \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}}$$

$$\cos \frac{x}{2} = \frac{1}{\sqrt{5}}$$

$$\sin \frac{x}{2} = \frac{2}{\sqrt{5}}$$

$$\tan \frac{x}{2} = 2$$

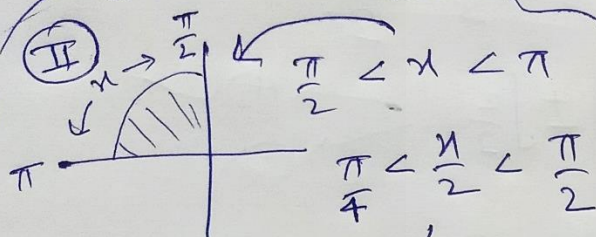
Q10 $\sin x = \frac{1}{4}$

II - quadrant

$$\sin \frac{x}{2} = \sqrt{\frac{4+\sqrt{15}}{8}}$$

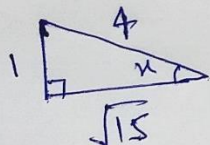
$$\cos \frac{x}{2} = \sqrt{\frac{4-\sqrt{15}}{8}}$$

$$\tan \frac{x}{2} = 4 + \sqrt{15}$$



$x \rightarrow \text{II}$
 $\frac{x}{2} \rightarrow \text{I}$

$\frac{P}{H} = \sin x = \frac{1}{4}$ $(x \rightarrow \text{II})$



$$\cos x = -\frac{\sqrt{15}}{4}$$

$$4 + \sqrt{15} = \sqrt{\frac{(4 + \sqrt{15})^2}{16 - 15}}$$

$$2 \cos^2 \frac{x}{2} = 1 + \cos x = 1 + \left(-\frac{\sqrt{15}}{4}\right)$$

$$\cos^2 \frac{x}{2} = \frac{4 - \sqrt{15}}{8}$$

$$\cos \frac{x}{2} = + \sqrt{\frac{4 - \sqrt{15}}{8}}$$

I-quadrant

$$2 \sin^2 \frac{x}{2} = 1 - \cos x = 1 + \frac{\sqrt{15}}{4}$$

$$\Rightarrow 2 \sin^2 \frac{x}{2} = \frac{4 + \sqrt{15}}{4}$$

$$\Rightarrow \sin^2 \frac{x}{2} = \frac{4 + \sqrt{15}}{8}$$

$$\sin \frac{x}{2} = + \sqrt{\frac{4 + \sqrt{15}}{8}}$$

(I)

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\sqrt{\frac{4 + \sqrt{15}}{8}}}{\sqrt{\frac{4 - \sqrt{15}}{8}}} = \sqrt{\frac{4 + \sqrt{15}}{4 - \sqrt{15}}} = \sqrt{\frac{4 + \sqrt{15}}{4 - \sqrt{15}} \times \frac{4 + \sqrt{15}}{4 + \sqrt{15}}}$$

All Useful Links:

- **YouTube Channel : Toppers Village** (Get full and free video lectures of Class 10 & 11 Maths) - <https://www.youtube.com/toppersvillage>
- **Website :** www.toppersvillage.com
- **Instagram :** <https://www.instagram.com/toppersvillage/>
- **Facebook Page:** www.facebook.com/ToppersVillage/
- **Facebook Group:** <https://www.facebook.com/groups/ToppersVillage>